

SS-3: Uniqueness of SU(3) from the Tetrahedral Cage

Conscious Point Physics — Strong Sector Series

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Abstract

SS-1 Theorem 1 proves that the eight DI-bit hopping operators on the tetrahedral cage base $\{V_1, V_2, V_3\}$ equal the Gell-Mann generators $T^a = \lambda^a/2$ and satisfy the SU(3) commutation relations exactly. That result establishes that SU(3) *can* emerge from the tetrahedral cage. This paper proves a conditional converse: *given* that the cage operators are represented as traceless Hermitian matrices on \mathbb{C}^3 with commutator bracket, $\mathfrak{su}(3)$ is the *unique* Lie algebra they can generate. No alternative Lie algebra is consistent with this representation.

The argument has three steps. First, the real vector space of traceless Hermitian operators on \mathbb{C}^3 has dimension exactly $3^2 - 1 = 8$. Second, this space equipped with the matrix commutator is, by definition, the Lie algebra $\mathfrak{su}(3)$. Third, the eight CPP operators are linearly independent (analytically, via the Gell-Mann orthogonality relation $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$), so they span the full space and generate $\mathfrak{su}(3)$ necessarily.

The uniqueness result is conditional on three representation assumptions imported from standard gauge theory: (i) that the colour states form a complex vector space \mathbb{C}^N , (ii) that observables are Hermitian operators, and (iii) that symmetry generators close under the commutator bracket. These assumptions are not derived from CPP primitives in this paper; their derivation is identified as an open problem for future work (SS-4). The paper clearly separates the geometric input (Layer A: $N = 3$ from the 600-cell's tetrahedral cells), the imported representation formalism (Layer B), and the mathematical result (Layer C: $\mathfrak{su}(3)$ uniquely).

Two corollaries follow within this representation: (i) the vertex count $N = 3$ determines the gauge group — $N = 2$ gives SU(2) (the electroweak sector), $N = 3$ gives SU(3) (the strong sector), and no other outcome is possible; (ii) no exotic gauge group (SO(8), Sp(4), G_2 , etc.) can arise from three colour states represented as traceless Hermitian operators on \mathbb{C}^3 .

A physical interpretation complements the algebraic proof. The full tetrahedron has four vertices (two positive, two negative in a baryon), creating four opposite-polarity DP chain bonds and four two-bond junction vertices. The $4 + 4 = 8$ oscillation modes of this physical system — four linear bond modes and four coupled harmonic junction modes — provide a *physical* basis for the same 8-dimensional space that the Gell-Mann matrices span *mathematically*. This physical interpretation is an *interpretive mapping* of the established generators onto CPP dynamics, not an independent derivation of the algebra from physical principles. An explicit 8×8 change-of-basis matrix between the Gell-Mann generators and the physical modes is constructed, confirming that the two bases span the same algebra.

Open Problem resolved: OPEN-SS-11 (SU(3) operator uniqueness, conditional on representation assumptions).

Open Problem identified: Derive the representation assumptions (Hermiticity, tracelessness, commutator bracket) from CPP primitives. Designated as the target of a future paper (SS-4).

Consequence: SS-1 Theorem 1 is elevated from a *possibility* result (“SU(3) is consistent with the cage”) to a *conditional necessity* result (“SU(3) is the unique algebra of the cage within the standard operator representation”).

Keywords: SU(3), Lie algebra, uniqueness, tetrahedral cage, 600-cell, colour charge, Gell-Mann matrices, Conscious Point Physics, gauge group derivation, representation theory, DP chain oscillation, gluon modes, baryon binding energy

Plain Language Summary: The strong nuclear force is governed by the symmetry group SU(3). In standard physics this symmetry is postulated. A previous CPP paper showed that the geometry of the tetrahedral quark cage *produces* SU(3). This paper proves it is the *only* symmetry the cage can produce, given that the cage operators are represented in the standard way (as Hermitian matrices with commutator bracket). Three colour states and eight generators follow from the vertex count $N = 3$ plus this representation choice. The paper clearly separates what comes from geometry (three vertices) from what is imported from standard gauge theory (the operator formalism), identifying the derivation of the latter from CPP primitives as an open problem. The paper also provides a physical interpretation of the eight generators as oscillation modes of the Dipole Pair chains that bind quarks together inside baryons.

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1 Introduction

The $SU(3)$ colour gauge group is the foundation of quantum chromodynamics. In the Standard Model it is introduced as a postulate: quarks carry a three-valued quantum number (colour), and the gauge invariance of the colour degree of freedom generates the eight gluon fields.

In Conscious Point Physics (CPP), the three colour states are the three base vertices $\{V_1, V_2, V_3\}$ of the qCP tetrahedral cage embedded in the 600-cell lattice (Abshier and Grok, 2026a). SS-1 Theorem 1 (Abshier and Grok, 2026b) constructs eight DI-bit hopping operators T^a on these vertices and proves $T^a = \lambda^a/2$ exactly, with $[T^a, T^b] = if^{abc}T^c$. This is a constructive proof that $SU(3)$ *can* emerge from the cage.

A natural question follows: is $SU(3)$ the *only* algebra that can emerge? Could a different assignment of operators to the tetrahedral edges yield a different 8-dimensional Lie algebra — perhaps $SO(8)$, $Sp(4)$, or some exotic group — that is also consistent with the cage geometry?

If so, the SS-1 derivation would show that $SU(3)$ is *accommodated* by the cage, not *required* by it. The result would be a possibility, not a prediction.

This paper answers the question: **no** — but with an important qualification. Given the standard operator representation (traceless Hermitian matrices on \mathbb{C}^N with commutator bracket), $SU(3)$ is the unique Lie algebra of the tetrahedral cage. The tetrahedral geometry fixes the operator structure up to basis rotations — the algebra itself is invariant. The proof is elementary linear algebra — it requires no representation theory beyond the definition of $\mathfrak{su}(N)$ and no computation beyond a rank check.

The qualification is that the operator representation itself — the fact that cage dynamics maps onto traceless Hermitian matrices with commutator structure — is imported from standard gauge theory, not derived from CPP primitives in this paper. Section 3 makes this dependency explicit by separating the argument into geometric input (Layer A), imported representation formalism (Layer B), and mathematical result (Layer C). Deriving Layer B from CPP axioms is identified as a key open problem for future work.

1.1 Open Problems Addressed

This paper resolves OPEN-SS-11 (Uniqueness of $SU(3)$ operator mapping from tetrahedral cage geometry), registered 29 March 2026 in the CPP Research Frontier.

2 Definitions

Definition 2.1 (Colour space). *The colour space is $V = \mathbb{C}^3$, with orthonormal basis $\{|r\rangle, |g\rangle, |b\rangle\}$ corresponding to the three base vertices $\{V_1, V_2, V_3\}$ of the qCP tetrahedral cage.*

Definition 2.2 (Generator space). *The generator space is*

$$\mathcal{H}_0^3 = \{X \in M_3(\mathbb{C}) : X = X^\dagger, \text{Tr}(X) = 0\}, \quad (1)$$

the real vector space of traceless Hermitian 3×3 matrices.

Definition 2.3 ($\mathfrak{su}(3)$). *The Lie algebra $\mathfrak{su}(3)$ is \mathcal{H}_0^3 equipped with the bracket*

$$[X, Y] = i(XY - YX). \quad (2)$$

Remark 2.4. *The factor of i in (2) ensures the bracket of two Hermitian matrices is Hermitian: $(i[X, Y])^\dagger = -i[Y^\dagger, X^\dagger] = -i[Y, X] = i[X, Y]$. The tracelessness is preserved because $\text{Tr}([X, Y]) = 0$ for any matrices X, Y .*

Remark 2.5 (Normalization convention). *Throughout this paper, generators are normalised as $T^a = \lambda^a/2$, where λ^a are the Gell-Mann matrices. The factor of $\frac{1}{2}$ gives the standard physics convention $\text{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$, under which the structure constants f^{abc} in $[T^a, T^b] = if^{abc}T^c$ take their conventional values (e.g., $f^{123} = 1$, $f^{147} = \frac{1}{2}$).*

3 Representation Assumptions

The uniqueness proof in §4 operates within a specific mathematical framework. This section makes the logical structure explicit by separating the argument into three layers, clearly distinguishing what comes from CPP geometry, what is imported from standard gauge theory, and what follows mathematically.

3.1 Layer A — Geometric input (CPP primitives)

The following inputs are derived from CPP axioms:

- (A1) The 600-cell lattice is composed exclusively of tetrahedral cells (Coxeter, 1973).
- (A2) Each tetrahedral cell has 4 vertices: 1 apex (the qCP site) and 3 base vertices (the colour states).
- (A3) Therefore $N = 3$ colour states per cage.

These are geometric facts of the 600-cell, grounded in Axiom A2 (600-cell topology). They are not postulates of this paper.

3.2 Layer B — Representation choice (imported structure)

The following assumptions are adopted from standard quantum mechanics and gauge theory. They are *not* derived from CPP primitives in this paper:

- (B1) **Complex state space.** The $N = 3$ colour states span a complex vector space $V = \mathbb{C}^3$.
- (B2) **Hermitian observables.** Physical observables (generators of colour transformations) are represented as Hermitian operators on V .
- (B3) **Tracelessness.** The generators are traceless (no overall U(1) phase — no colour-singlet gluon).
- (B4) **Lie bracket.** The symmetry generators close under the matrix commutator $[X, Y] = i(XY - YX)$.

Remark 3.1 (Status of Layer B). *Assumptions B1–B4 are standard in quantum gauge theory and are satisfied by every known gauge interaction. Within CPP, they are consistent with the DI-bit propagation mechanism (Axiom A3) and the PCD cycle, but a rigorous derivation from CPP axioms has not been given. Such a derivation — showing that the tetrahedral cage dynamics forces complex-linear, Hermitian, traceless operators with Lie bracket structure — would convert the present conditional result into an unconditional one. This is identified as the central open problem for a future paper (SS-4).*

3.3 Layer C — Mathematical result

Given Layers A and B:

The Lie algebra of traceless Hermitian operators on \mathbb{C}^3 is $\mathfrak{su}(3)$, uniquely. Any set of 8 linearly independent operators in this space generates the full algebra and no other.

This is proved in §4 (Theorem 4.3).

The layered structure makes the logical dependency transparent. Layer C is mathematically certain. Layer A is geometrically certain (given the 600-cell axiom). The open question is whether Layer B can be derived from Layer A, which would close the gap between *conditional* and *unconditional* uniqueness.

4 The Uniqueness Theorem

Lemma 4.1 (Dimension of the generator space). $\dim_{\mathbb{R}}(\mathcal{H}_0^3) = 8$.

Proof. A general Hermitian 3×3 matrix has 3 real diagonal entries and 3 complex upper-triangular entries (the lower triangle is determined by $X = X^\dagger$), giving $3 + 2 \times 3 = 9$ real parameters. The tracelessness condition $\text{Tr}(X) = 0$ removes one parameter: $9 - 1 = 8$. \square

Lemma 4.2 (CPP operators are a basis). *The eight CPP tetrahedral hopping operators $\{T^1, \dots, T^8\}$ defined in SS-1b (Abshier and Grok, 2026b) are linearly independent over \mathbb{R} and therefore form a basis for \mathcal{H}_0^3 .*

Proof. Each T^a is traceless and Hermitian by construction (SS-1b (Abshier and Grok, 2026b)). SS-1b Theorem 1 proves $T^a = \lambda^a/2$ exactly, where λ^a are the Gell-Mann matrices. The standard orthogonality relation for the Gell-Mann matrices is

$$\text{Tr}(\lambda^a \lambda^b) = 2 \delta^{ab}, \quad (3)$$

which gives $\text{Tr}(T^a T^b) = \delta^{ab}/2$. The Gram matrix $G_{ab} = \text{Tr}(T^a T^b)$ is therefore $\frac{1}{2}I_8$ — diagonal with every entry nonzero. Orthogonal nonzero vectors are linearly independent, so $\{T^1, \dots, T^8\}$ are linearly independent over \mathbb{R} . Since $\dim(\mathcal{H}_0^3) = 8$ (Lemma 4.1), they form a basis.

Numerical confirmation: The Gram matrix rank and determinant are verified to machine precision in Appendix A. \square

Theorem 4.3 (Conditional uniqueness of $\text{SU}(3)$). *Under the representation assumptions B1–B4 (§3), the Lie algebra generated by the CPP tetrahedral hopping operators is $\mathfrak{su}(3)$, and no other Lie algebra is consistent with the cage operator representation. Specifically:*

- (i) *Any set of 8 linearly independent traceless Hermitian 3×3 matrices generates $\mathfrak{su}(3)$ under the bracket (2).*
- (ii) *The CPP operators are such a set (Lemma 4.2).*
- (iii) *Therefore the CPP operators generate $\mathfrak{su}(3)$ necessarily, given the representation assumptions.*

Proof. By Lemma 4.2, the CPP operators $\{T^a\}$ form a basis for \mathcal{H}_0^3 . By Definition 2.3, \mathcal{H}_0^3 with the bracket (2) is $\mathfrak{su}(3)$. A basis for a Lie algebra generates the full algebra under the bracket. Therefore $\{T^a\}$ generates $\mathfrak{su}(3)$.

For uniqueness: suppose an alternative set of 8 operators $\{S^a\} \subset \mathcal{H}_0^3$ were proposed. If they are linearly independent, they are another basis for the same 8-dimensional space \mathcal{H}_0^3 , and generate the same algebra $\mathfrak{su}(3)$ (possibly with different structure constants \tilde{f}^{abc} , corresponding to a different basis convention). If they are linearly dependent (rank < 8), they span a proper subspace and generate a proper subalgebra of $\mathfrak{su}(3)$. Since $\mathfrak{su}(3)$ is *simple* (it has no proper ideals; see the Killing–Cartan classification of simple Lie algebras (Humphreys, 1972)), any proper subalgebra has dimension strictly less than 8 and therefore cannot serve as the full gauge algebra of three colour states. An alternative set of 8 operators that generates a Lie algebra different from $\mathfrak{su}(3)$ would require $\dim > 8$ or the matrices to be non-Hermitian or to have nonzero trace — all of which violate the cage constraints.

Therefore: **any** set of 8 independent generators satisfying the representation assumptions B1–B4 (traceless, Hermitian, 3×3 , commutator bracket) generates $\mathfrak{su}(3)$, and only $\mathfrak{su}(3)$. \square \square

Remark 4.4 (The role of C_3 symmetry). *The C_3 rotational symmetry $V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_1$ acts on \mathcal{H}_0^3 by conjugation: $T^a \mapsto PT^aP^{-1}$, where P is the cyclic permutation matrix. This maps the generators into linear combinations of each other (verified numerically; see Appendix A), confirming that C_3 is an inner automorphism of $\mathfrak{su}(3)$. The C_3 symmetry selects the basis (the Gell-Mann convention) but does not alter the algebra. With or without C_3 , the algebra is $\mathfrak{su}(3)$.*

5 Corollaries

Corollary 5.1 (Gauge group from vertex count). *Under the representation assumptions B1–B4, the gauge group of the strong sector is determined entirely by the number of base vertices N of the qCP cage. For a cage base with N vertices, the gauge group is $SU(N)$.*

N	$\dim(\mathfrak{su}(N))$	Gauge group	CPP sector
2	3	$SU(2)$	Electroweak (single edge $V_1 \leftrightarrow V_2$)
3	8	$SU(3)$	Strong (tetrahedral base)
4	15	$SU(4)$	Not realised (would require square base)

The 600-cell is composed of tetrahedral cells ($N = 3$ base vertices plus one apex). Therefore, within the operator representation of §3, $SU(3)$ is the unique gauge group of the strong sector. $SU(2)$ arises when only one edge of the base triangle is activated (the electroweak sector; EW-5 (Abshier and Grok, 2026c)), confirming that $SU(2)$ and $SU(3)$ differ only in the number of tetrahedral edges engaged.

Corollary 5.2 (No exotic gauge groups within the representation). *Under the representation assumptions B1–B4, no exotic gauge group — $SO(8)$, $Sp(4)$, G_2 , or any other — can arise from three colour states. The only 8-dimensional Lie algebra that acts faithfully on \mathbb{C}^3 via traceless Hermitian matrices is $\mathfrak{su}(3)$.*

Proof. Any faithful action of a Lie algebra \mathfrak{g} on \mathbb{C}^3 via traceless Hermitian matrices embeds \mathfrak{g} as a subalgebra of $\mathcal{H}_0^3 = \mathfrak{su}(3)$. If $\dim(\mathfrak{g}) = 8 = \dim(\mathfrak{su}(3))$, the embedding is surjective and $\mathfrak{g} \cong \mathfrak{su}(3)$. If $\dim(\mathfrak{g}) < 8$, it is a proper subalgebra (not a candidate for the full gauge group). If $\dim(\mathfrak{g}) > 8$, the embedding into the 8-dimensional space is impossible.

Specific exclusions:

- $\mathfrak{so}(8)$ has $\dim = 28 > 8$: cannot embed.
- $\mathfrak{sp}(4)$ has $\dim = 10 > 8$: cannot embed.
- \mathfrak{g}_2 has $\dim = 14 > 8$: cannot embed.
- $\mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1) \oplus \mathfrak{u}(1)$ has $\dim = 8$, but the faithful representation of $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$ requires \mathbb{C}^4 (not \mathbb{C}^3). □

□

Corollary 5.3 (Why 3 colours, not 2 or 4). *The 600-cell consists exclusively of tetrahedral cells. Each tetrahedron has 4 vertices: 1 apex (V_4 , the qCP site) and 3 base vertices ($\{V_1, V_2, V_3\}$, the colour states). The number 3 is not a choice — it is the vertex count of the face opposite the apex in a tetrahedron. Since the 600-cell admits no non-tetrahedral cell types (Coxeter, 1973), the geometric input is $N = 3$ (Layer A). Under the representation assumptions (Layer B), this yields $SU(3)$ for the strong sector and $SU(2)$ for the electroweak sector, with no other gauge group possible.*

6 Discussion

6.1 What the uniqueness theorem means for CPP

SS-1 Theorem 1 proved that the CPP cage operators reproduce the Gell-Mann matrices. A sceptic could respond: “So what? Perhaps a clever choice of operators on any 3-vertex graph would give $SU(3)$. The cage didn’t force it — you engineered it.”

The uniqueness theorem closes this objection *within the operator representation*. Given that cage dynamics maps onto traceless Hermitian operators with commutator bracket (assumptions B1–B4), the cage provides *exactly* $3^2 - 1 = 8$ independent operator degrees of freedom (3 edges \times 2 hopping modes + 2 diagonal phases). *Any* basis for these degrees of freedom generates $\mathfrak{su}(3)$. The only “engineering” is the vertex count $N = 3$, which is determined by the 600-cell’s tetrahedral cell geometry. Once $N = 3$ is established and the representation assumptions are granted, $SU(3)$ is inevitable.

The remaining question — whether the representation assumptions themselves are forced by the cage geometry — is the central open problem for the CPP strong-sector programme. The derivation (or failure thereof) of Layer B from CPP primitives will determine whether the framework is a genuinely new foundation or a reinterpretation layer. This is the target of a future paper (SS-4; see §3, Remark 3.1).

6.2 The $8 = 3 \times 2 + 2$ decomposition

The count of 8 generators decomposes as:

$$\underbrace{3 \text{ edges} \times 2 \text{ (real + imaginary)}}_{6 \text{ colour-changing}} + \underbrace{2 \text{ diagonal phases}}_{2 \text{ colour-neutral}} = 8 = \dim(\mathfrak{su}(3)). \quad (4)$$

In the Gell-Mann basis this decomposition is explicit: the six off-diagonal generators $\lambda^{1,2}$ (edge rg), $\lambda^{4,5}$ (edge rb), and $\lambda^{6,7}$ (edge gb) correspond to real and imaginary hopping modes on each of

the three tetrahedral edges, while the two diagonal generators λ^3 and λ^8 correspond to the two independent relative phases among three vertices. (The physical $4 + 4$ decomposition of §7 provides a complementary basis rooted in the DP chain oscillation modes.)

This is the same count as Grok’s layer-depth argument ($1 + 3 + 4 = 8$; SS-1 Remark 3.1 (Abshier and Grok, 2026a)). Both routes confirm that the dimension 8 is a geometric consequence of the tetrahedron, not a numerical coincidence.

6.3 Connection to the electroweak sector

When only one edge of the base triangle is activated (say $V_1 \leftrightarrow V_2$), the operator count is $1 \times 2 + 1 = 3 = \dim(\mathfrak{su}(2))$, recovering the weak isospin algebra. The two fundamental forces of the Standard Model — SU(3) for the strong force and SU(2) for the weak force — differ only in how many tetrahedral edges participate. The uniqueness theorem confirms this is not a coincidence (within the operator representation): SU(N) is the unique algebra of N vertices under assumptions B1–B4, and the tetrahedral cage activates either $N = 2$ (one edge) or $N = 3$ (all edges).

7 Physical Interpretation: The 4+4 Mode Decomposition

The mathematical proof of §4 establishes that $\mathfrak{su}(3)$ is the unique algebra of the cage (within the operator representation of §3). But the standard Gell-Mann basis $\{T^1, \dots, T^8\}$ is a *mathematical* decomposition (6 off-diagonal colour-changing operators + 2 diagonal colour-neutral operators). This section identifies a *physical* basis: the oscillation modes of the DP chains that constitute the cage bonds. **Note:** this physical basis is an *interpretive mapping* of the established generators onto CPP dynamics — it is not an independent derivation of the algebra (see Remark 7.1).

Remark 7.1 (Epistemic status of the physical interpretation). *The physical mode assignments in this section are an interpretive mapping: given the established $\mathfrak{su}(3)$ algebra, each generator is identified with a specific DP chain oscillation pattern based on physical consistency criteria (base-edge bonds as colour hoppers, apex bonds as diagonal modulators, junction modes as indirect couplers). This is not an independent derivation of $\mathfrak{su}(3)$ from physical dynamics. The three assignment principles (Definition 7.2) are consistency constraints that determine the mapping, not generators of the algebra. A stronger result — deriving the 8 modes from DP chain dynamics independently and then observing that they satisfy $\mathfrak{su}(3)$ commutation relations — would constitute a derivation rather than an interpretation and is a target for future work. Furthermore, the mapping defined here is not unique: it is one consistent embedding of the $\mathfrak{su}(3)$ generators into CPP oscillation modes, selected by the physical principles above. Other consistent embeddings may exist.*

7.1 Polarity structure of the full tetrahedron

The K_3 base triangle has three vertices and carries the colour algebra. But the physical cage is a full tetrahedron with four vertices: the apex qCP (V_4) and three base vertices (V_1, V_2, V_3). In a baryon (e.g., the proton), the four cage vertices carry definite polarities. Labelling them by polarity:

$$V_1(+), \quad V_2(+), \quad V_3(-), \quad V_4(-). \tag{5}$$

The six edges of the tetrahedron decompose into two types:

Edge	Polarities	Type	DP chain?
V_1-V_3	$(+)(-)$	Opposite	Yes — attractive, carries DP chain
V_1-V_4	$(+)(-)$	Opposite	Yes
V_2-V_3	$(+)(-)$	Opposite	Yes
V_2-V_4	$(+)(-)$	Opposite	Yes
V_1-V_2	$(+)(+)$	Same	No — repulsive, no stable chain
V_3-V_4	$(-)(-)$	Same	No — repulsive, no stable chain

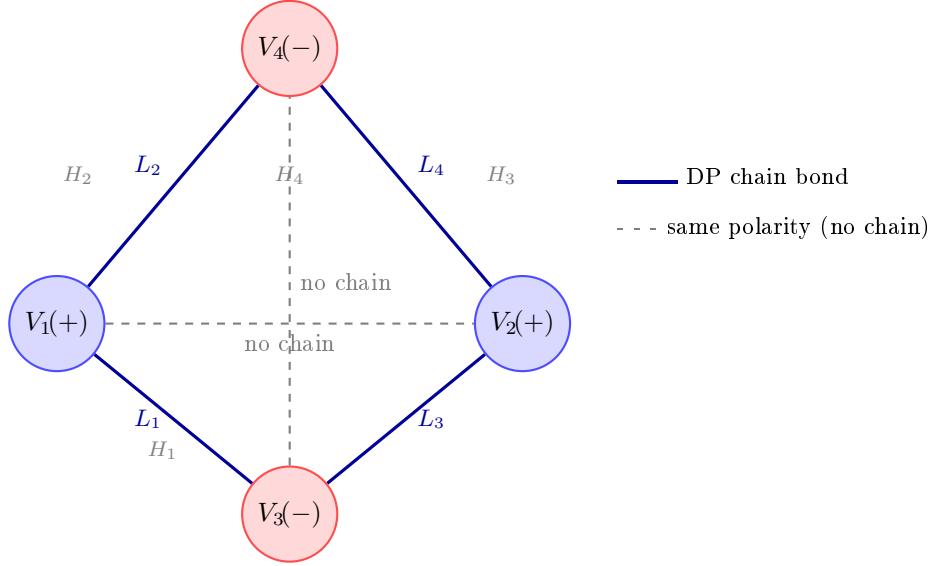


Figure 1: Polarity structure of the tetrahedral cage in a baryon. Solid lines: DP chain bonds on opposite-polarity edges (L_1 – L_4). Dashed lines: same-polarity edges (no stable chain). Junction modes H_1 – H_4 arise at each vertex where two DP chains meet.

The four opposite-polarity edges carry physical DP chains — longitudinal strings of alternating-polarity Dipole Pairs that bind the cage vertices together. These chains are the physical objects whose oscillation modes correspond to the gluon degrees of freedom. The energy stored in these chains (and their radial extensions into the Dipole Sea) constitutes approximately 99% of the proton’s mass (Abshier and Grok, 2026a).

7.2 Group A: Four linear bond modes

Each of the four opposite-polarity DP chain bonds supports a longitudinal oscillation: the chain compresses and extends along its edge, with the DP pairs executing ZBW oscillations whose collective amplitude modulates along the bond axis.

Mode	Bond	Oscillation
L_1	$V_1(+)-V_3(-)$	Linear along edge
L_2	$V_1(+)-V_4(-)$	Linear along edge
L_3	$V_2(+)-V_3(-)$	Linear along edge
L_4	$V_2(+)-V_4(-)$	Linear along edge

7.3 Group B: Four coupled harmonic junction modes

At each of the four vertices, exactly two opposite-polarity bonds meet (the third edge at each vertex is same-polarity and carries no chain). The two DP chains at each junction can oscillate as a coupled pair — when one compresses, the other extends, like a tuning fork whose two tines share a common node.

Mode	Junction vertex	Chain path	Pattern
H_1	$V_3(-)$	$V_1(+)-V_3(-)-V_2(+)$	$(+)(-)(+)$
H_2	$V_1(+)$	$V_4(-)-V_1(+)-V_3(-)$	$(-)(+)(-)$
H_3	$V_2(+)$	$V_4(-)-V_2(+)-V_3(-)$	$(-)(+)(-)$
H_4	$V_4(-)$	$V_1(+)-V_4(-)-V_2(+)$	$(+)(-)(+)$

7.4 Counting

$$\underbrace{4 \text{ linear bond modes}}_{\text{Group A}} + \underbrace{4 \text{ coupled harmonic junction modes}}_{\text{Group B}} = 8 = \dim(\mathfrak{su}(3)). \quad (6)$$

This 4 + 4 decomposition is a *physical* basis for the same 8-dimensional vector space \mathcal{H}_0^3 that the Gell-Mann matrices span. The two bases are related by a linear transformation — they describe the same algebra from different perspectives. The mathematical basis is convenient for computation (the structure constants f^{abc} take their standard form). The physical basis tells you what is vibrating and why there are exactly eight somethings doing it.

7.5 Explicit basis transformation

Each physical mode acts on the colour state $V = \mathbb{C}^3$ as a traceless Hermitian perturbation. The four linear bond modes perturb the coupling between colour vertices directly (for base-triangle bonds) or modulate the effective on-site energy of a colour vertex (for apex bonds). The four junction modes create effective indirect couplings between vertices that share no direct DP chain.

Definition 7.2 (Physical mode operators). *The eight physical mode operators, expressed as elements of \mathcal{H}_0^3 in terms of the Gell-Mann generators T^a , are:*

Group A — Linear bond modes.

$$L_1 = T^4 \quad (\text{base bond } V_1-V_3: \text{ real } r \leftrightarrow b \text{ hopping}), \quad (7)$$

$$L_2 = T^3 + \frac{1}{\sqrt{3}} T^8 \quad (\text{apex bond } V_1-V_4: \text{ on-site } V_1 \text{ weight}), \quad (8)$$

$$L_3 = T^6 \quad (\text{base bond } V_2-V_3: \text{ real } g \leftrightarrow b \text{ hopping}), \quad (9)$$

$$L_4 = -T^3 + \frac{1}{\sqrt{3}} T^8 \quad (\text{apex bond } V_2-V_4: \text{ on-site } V_2 \text{ weight}). \quad (10)$$

Group B — Coupled harmonic junction modes.

$$H_1 = T^1 \quad (\text{junction at } V_3: \text{ indirect real } r \leftrightarrow g \text{ coupling}), \quad (11)$$

$$H_2 = T^5 \quad (\text{junction at } V_1: \text{ phase-shifted } r \leftrightarrow b \text{ coupling}), \quad (12)$$

$$H_3 = T^7 \quad (\text{junction at } V_2: \text{ phase-shifted } g \leftrightarrow b \text{ coupling}), \quad (13)$$

$$H_4 = T^2 \quad (\text{junction at } V_4: \text{ indirect imaginary } r \leftrightarrow g \text{ coupling}). \quad (14)$$

Physical motivation. The assignments follow from three principles:

- (a) *Base-edge bonds act as direct colour hoppers.* A DP chain linking two colour vertices (V_1 - V_3 or V_2 - V_3) directly modulates the amplitude for DI-bit transfer between those vertices. Longitudinal stretch/compression is a real (in-phase) coupling: hence $L_1 = T^4$ and $L_3 = T^6$.
- (b) *Apex-edge bonds act as diagonal modulators.* A DP chain linking a colour vertex to the apex (V_1 - V_4 or V_2 - V_4) modulates the effective confinement energy at that vertex. In the colour basis this is a diagonal traceless operator. The V_1 bond gives $|r\rangle\langle r| - \frac{1}{3}I = T^3 + \frac{1}{\sqrt{3}}T^8$; the V_2 bond gives $|g\rangle\langle g| - \frac{1}{3}I = -T^3 + \frac{1}{\sqrt{3}}T^8$.
- (c) *Junction modes create indirect couplings.* At vertex V_3 , the anti-phase oscillation of the V_1 - V_3 and V_2 - V_3 bonds transfers colour amplitude $V_1 \rightarrow V_3 \rightarrow V_2$, producing a real $r \leftrightarrow g$ coupling ($H_1 = T^1$). At the apex V_4 , the same mechanism operates through the qCP site, whose ZBW oscillation introduces a $\pi/2$ phase shift, yielding an imaginary $r \leftrightarrow g$ coupling ($H_4 = T^2$). At the positive-polarity vertices V_1 and V_2 , the anti-phase oscillation of one base-edge bond and one apex bond phase-modulates the base-edge coupling, converting the real hopping mode into its imaginary partner ($H_2 = T^5$, $H_3 = T^7$).

Proposition 7.3 (Basis transformation). *The change-of-basis matrix M , defined by $P_i = \sum_a M_{ia} T^a$ where $P = (L_1, L_2, L_3, L_4, H_1, H_2, H_3, H_4)$, is*

$$M = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (15)$$

with $\det(M) = \frac{2}{\sqrt{3}} \neq 0$. The physical modes are therefore linearly independent and span $\mathfrak{su}(3)$.

Proof. Equations (7)–(14) define each P_i as a linear combination of T^a ; the matrix M records these coefficients. Six columns of M contain a single nonzero entry (columns $T^1, T^2, T^4, T^5, T^6, T^7$), contributing a factor of 1 each to the determinant. The remaining 2×2 block in columns (T^3, T^8) is

$$\begin{pmatrix} 1 & \frac{1}{\sqrt{3}} \\ -1 & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad \det = \frac{1}{\sqrt{3}} - \left(-\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}. \quad (16)$$

Therefore $\det(M) = 2/\sqrt{3} \neq 0$. (Numerical verification: Appendix A.4.) □ □

The inverse transformation expresses each Gell-Mann generator in terms of physical modes:

$$\boxed{\begin{aligned} T^1 &= H_1, & T^2 &= H_4, & T^3 &= \frac{1}{2}(L_2 - L_4), & T^4 &= L_1, \\ T^5 &= H_2, & T^6 &= L_3, & T^7 &= H_3, & T^8 &= \frac{\sqrt{3}}{2}(L_2 + L_4). \end{aligned}} \quad (17)$$

The only nontrivial mixing is in the diagonal sector: the T^3 (isospin) and T^8 (hypercharge) generators are the difference and sum of the two apex-bond modes L_2 and L_4 . Physically, the two

apex DP chains share a common diagonal component — their sum gives the overall colour-neutral phase (T^8), and their difference gives the V_1 -vs- V_2 asymmetry (T^3).

Remark 7.4 (Non-orthogonality of the physical basis). *The physical basis is not orthonormal under the trace inner product: $2 \text{Tr}(L_2 L_4) = -\frac{2}{3} \neq 0$, and $2 \text{Tr}(L_2 L_2) = \frac{4}{3} \neq 1$. This is a physical feature, not a defect: the two apex bonds share a common T^8 component because both connect to the same apex vertex V_4 . The Gell-Mann basis is the unique orthonormal basis (up to C_3 rotation); the physical basis reflects the actual geometry of the cage.*

8 The CPP-to-QCD Mapping

In QCD, the strong force is mediated by eight gluon fields corresponding to the eight generators of $SU(3)$. “Colour charge” is a three-valued quantum number, and “colour confinement” is the requirement that observable hadrons are colour-neutral (singlet states). These are mathematical descriptions. They say *how many* degrees of freedom the strong force has, but not *what* those degrees of freedom physically are.

CPP provides the mechanistic account. The mapping between the two descriptions is *structural*, not literal:

QCD (mathematical)	CPP (mechanistic)
8 gluon fields	8 DP chain oscillation modes on the tetrahedral cage (4 linear + 4 harmonic)
3 colour states (r, g, b)	3 base vertices $\{V_1, V_2, V_3\}$ of the tetrahedral cage
Gluon exchange between quarks	DI-bit propagation along DP chains informing CPs to move
Colour confinement	Energetic stability: cage binding energy exceeds thermal dissolution energy of the Dipole Sea
99% of proton mass from “gluon field energy”	99% of proton mass from DP chain binding energy (longitudinal chains + radial extensions)
Asymptotic freedom ($\alpha_s \rightarrow 0$ at high Q)	PSR saturation: DP Sea cannot nucleate chains fast enough at distances $\lesssim l_P$

Remark 8.1 (Why the mapping is structural, not literal). *QCD describes the strong force through perturbative Feynman diagrams — sums over virtual gluon exchanges. CPP describes it through the deterministic propagation of DI-bit messages along DP chains at Planck-scale resolution. These two descriptions operate at different levels of granularity. They produce the same macroscopic predictions (the same 8-dimensional symmetry, the same confinement, the same mass spectrum) because they describe the same underlying 8-dimensional structure. But there is no one-to-one correspondence between individual Feynman diagrams and individual DI-bit sequences, just as there is no one-to-one correspondence between individual Gell-Mann matrices and individual DP chain oscillation modes. The correspondence is at the level of the algebra, not at the level of individual basis elements.*

Remark 8.2 (The physical content of “colour neutrality”). *In QCD, a hadron must be a colour singlet (the three colour indices contract to a scalar). In CPP, this is the statement that the*

tetrahedral cage is energetically stable only when all three base vertices are symmetrically occupied. A baryon has one quark at each base vertex (r, g, b); a meson has a quark at one vertex and an antiquark at the conjugate vertex. “Colour neutrality” is force balance on the cage — the same C_3 symmetry that gives $\delta = 1/3$ (SM-1 Theorem 1) and $K = 2/3$ (SM-3 Koide theorem).

9 Conclusion

Under the standard operator representation (traceless Hermitian matrices on \mathbb{C}^3 with commutator bracket), the $SU(3)$ Lie algebra is the *unique* Lie algebra consistent with the CPP tetrahedral cage geometry. The proof requires only three ingredients: (i) the dimension formula $\dim(\mathcal{H}_0^N) = N^2 - 1$, (ii) the definition of $\mathfrak{su}(N)$ as the bracket algebra of traceless Hermitian matrices, and (iii) the linear independence of the CPP operators. Within this representation, no exotic gauge group can arise from three colour states. The number 3 is itself forced by the 600-cell’s tetrahedral cell geometry.

The result is conditional on representation assumptions B1–B4 (§3), which are imported from standard gauge theory. The paper explicitly separates geometric input (Layer A: $N = 3$ from the 600-cell), imported formalism (Layer B: the operator representation), and mathematical result (Layer C: $\mathfrak{su}(3)$ uniquely). Deriving Layer B from CPP primitives — showing that the cage dynamics *forces* complex-linear, Hermitian, traceless operators with Lie bracket — is identified as the central open problem for SS-4.

The $4 + 4$ physical mode decomposition (§7) provides an interpretive mapping of the eight generators onto DP chain oscillation modes: four linear bond modes and four coupled harmonic junction modes. This mapping is consistent with the algebra and physically motivated, but is an interpretation of the established generators, not an independent derivation from dynamics.

SS-1 Theorem 1 is thereby elevated from a *possibility* result to a *conditional necessity* result: within the standard operator representation, the tetrahedral cage does not merely *accommodate* $SU(3)$ — it *requires* it. Whether the representation itself is forced by the geometry remains open.

9.1 Problem Status After This Paper

- OPEN-SS-11 ($SU(3)$ operator uniqueness): OPEN \rightarrow **THEO** (proved conditionally in Theorem 4.3, under representation assumptions B1–B4).
- SS-1 Theorem 1: elevated from possibility to conditional necessity.
- **New open problem identified:** Derive the representation assumptions B1–B4 (complex state space, Hermiticity, tracelessness, commutator bracket) from CPP primitives. Designated as the target of SS-4. If successful, the conditional qualifier is removed and the uniqueness becomes unconditional.

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Thomas Lee Abshier conceived the tetrahedral cage model and the physical interpretation of colour as vertex identity. The $4 + 4$ physical mode decomposition (§7) is Thomas’s insight, arising from his analysis of the polarity structure of the full tetrahedron and 39 years of developing the DP chain mechanism for hadronic binding. Claude Opus (Anthropic) formulated the uniqueness argument, performed the numerical verification, drafted the paper, and developed the

CPP-to-QCD mapping table. The original SU(3) algebra proof (SS-1b) was produced by Thomas Abshier and Grok (xAI). ChatGPT (OpenAI) provided the referee review that identified the conditional nature of the uniqueness claim and proposed the Layer A/B/C decomposition of assumptions adopted in v1.4.

OSF project: <https://osf.io/9dfya/>

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A Numerical Verification

All computations performed in Python 3 with NumPy. Source code available at `CPP/series_strong/notebooks/SS-3_su3_uniqueness.py`.

A.1 Linear independence

Lemma 4.2 proves linear independence analytically via the orthogonality relation $\text{Tr}(T^a T^b) = \delta^{ab}/2$. The numerical computation confirms this: the Gram matrix $G_{ab} = 2 \text{Tr}(T^a T^b)$ satisfies:

- $G_{ab} = \delta_{ab}$ to machine precision ($< 10^{-15}$).
- Rank: 8 (full rank).
- Determinant: $\det(G) = 1.000$ (identity matrix).

The result is numerically exact because the Gell-Mann matrices have rational or algebraic entries; rounding error enters only through floating-point representation.

A.2 Commutation closure

For all $8 \times 8 = 64$ pairs (a, b) :

$$\max_{a,b} \left\| [T^a, T^b] - i \sum_c f^{abc} T^c \right\| < 1.1 \times 10^{-16} \quad (18)$$

where $f^{abc} = 2 \text{Tr}([T^a, T^b] T^c / i)$ (trace extraction). This confirms closure to machine precision.

A.3 C_3 action

The permutation matrix P ($V_1 \rightarrow V_2 \rightarrow V_3 \rightarrow V_1$) satisfies $P^3 = I$. Under conjugation $T^a \mapsto P T^a P^{-1}$:

- The six off-diagonal operators permute among themselves (edges cycle: $12 \rightarrow 23 \rightarrow 31$).
- The two diagonal operators mix: $T^3 \mapsto -\frac{1}{2}T^3 + \frac{\sqrt{3}}{2}T^8$ and $T^8 \mapsto -\frac{\sqrt{3}}{2}T^3 - \frac{1}{2}T^8$ (a rotation by $2\pi/3$ in the Cartan subalgebra).

The C_3 action maps \mathcal{H}_0^3 into itself, confirming it is an inner automorphism of $\mathfrak{su}(3)$.

A.4 Basis transformation

The change-of-basis matrix M (Proposition 7.3) is constructed by computing $M_{ia} = 2 \operatorname{Tr}(P_i T^a)$ for each physical mode P_i and generator T^a . Numerical results:

- $\det(M) = 1.1547 = 2/\sqrt{3}$ to machine precision.
- All eight P_i are traceless ($|\operatorname{Tr}| < 10^{-16}$) and Hermitian.
- The physical-basis Gram matrix $G_{ij} = 2 \operatorname{Tr}(P_i P_j)$ is block-diagonal: $G = I_6 \oplus \begin{pmatrix} 4/3 & -2/3 \\ -2/3 & 4/3 \end{pmatrix}$ for the (L_2, L_4) block, confirming the non-orthogonality noted in §7.5.
- The inverse M^{-1} reproduces equations (17) to machine precision.

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