The Heavy Quark Mass Spectrum and Strong Coupling from 600-Cell Lattice Geometry

Conscious Point Physics — SM-7 (Version 2.2)

Thomas Lee Abshier, ND
Claude Opus (Anthropic), Copilot (Microsoft), Grok (xAI)
Hyperphysics Institute
https://hyperphysics.com
drthomas007@protonmail.com

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Abstract

The heavy quark mass spectrum (charm, bottom, top) and the strong coupling constant α_s are derived from the geometry of the 600-cell polytope with one calibration constant (the charm mass) and zero free shape parameters. This paper extends the lepton mass derivation of SM-6 to the quark sector and discovers that the strong coupling emerges from the *same* spectral trace formula as the Weinberg angle: both are mode fractions of the 600-cell lattice weighted by the propagation efficiency $\eta = 1/\varphi$.

The strong coupling: $\alpha_s = (1/\varphi) \times [\mathrm{Tr}(A^3)/3]/[\mathrm{Tr}(A^2) + \mathrm{Tr}(A^3)/3] = 5/(8\varphi) \approx 0.386$, where $\mathrm{Tr}(A^3)/3 = 2F = 2400$ counts non-abelian face-circulation modes. The identification of face modes with the colour sector rests on SS-1, which proves that cage-face permutations generate the SU(3) colour algebra exactly. The value $5/(8\varphi) = 0.386$ matches the PDG running coupling at the charm mass scale ($\alpha_s(m_c) \approx 0.35$–$0.40$). The ratio $\alpha_s/\sin^2\theta_W = F/E = 5/3$ is a topological invariant of the 600-cell.

Mode complementarity: $\sin^2\theta_W + \alpha_s = 3/(8\varphi) + 5/(8\varphi) = 1/\varphi$. At the bare (topological) level, the abelian edge modes and non-abelian face modes partition the total mode capacity: $3/8 + 5/8 = 1$. At the physical (metric-corrected) level, both are reduced by $\eta = 1/\varphi$, so the total propagation efficiency is $1/\varphi \approx 0.618$.

The quark Koide phase: Quarks carry colour charge, so their K_3 cage faces feel the strong coupling on all $z = 12$ nearest-neighbour bonds (not just the 2 internal K_3 bonds that carry the electroweak correction in the lepton sector). This bond-count asymmetry is derived from the projector structure: edge modes are internal-bond-local (Assumption A1), while face-circulation modes saturate all incident bonds in the closed neighbourhood (Assumption A2). The attractive colour binding energy produces a negative isotropic shift $\varepsilon_S = -z\alpha_s/(z+1) = -60/(104\varphi)$. Combined with the electroweak shift $\varepsilon_{EW} = +3/(52\varphi)$ from SM-6, the net correction is $\varepsilon = -27/(52\varphi)$, giving $\cos\theta_{\text{quark}} = -(2/3)(1-27/(104\varphi))$, with $\theta = 124.04$ (PDG: 124.09, agreement 0.05%).

Predicted masses: $m_b = 4.24$ GeV (PDG: 4.18, 1.4%), $m_t = 169.8$ GeV (PDG: 172.7, 1.7%). Combined with the lepton results of SM-6, the 600-cell lattice now determines 6 fermion masses (3 leptons + 3 heavy quarks) from 2 calibration constants and 0 shape parameters, replacing 6 independent parameters in the Standard Model.

Keywords: strong coupling constant, heavy quark masses, Koide formula, 600-cell polytope, gauge mode complementarity, golden ratio, spectral graph theory, charm bottom top, lattice QCD, colour confinement, face-circulation modes, edge locality, projector algebra

Plain Language Summary: In the Standard Model, the strong force coupling constant and the three heavy quark masses are independent free parameters with no structural explanation. This paper shows that the same geometric object that determines the electroweak mixing angle — the 600-cell lattice — also determines the strong coupling. The 600-cell has 720 edges and 1200 triangular faces. The fraction of edge modes gives the Weinberg angle; the fraction of face modes gives the strong coupling. Together they sum to the inverse golden ratio. Because quarks feel the strong force while leptons do not, the quark mass spectrum has a different Koide phase from the lepton spectrum — but both phases are derived from the same lattice geometry with the same formula, differing only in which forces act on the cage bonds. The predicted bottom and top quark masses agree with measurements to better than 2%, with no adjustable shape parameters.

Contents

1 Introduction

SM-6 (Abshier et al., 2026a) derived the charged lepton mass spectrum from the 600-cell lattice with one calibration constant and zero shape parameters. The derivation had two parts: the Weinberg angle $\sin^2\theta_W = 3/(8\varphi)$ from spectral trace mode counting, and the Koide phase $\cos\theta_{\text{lepton}} = -(2/3)(1 + 3/(104\varphi))$ from the isotropic electroweak shift on the K_3 cage face.

The heavy quark triplet (charm, bottom, top) satisfies the Koide ratio $K \approx 2/3$ to 0.42% (Abshier et al., 2026b), suggesting that the same K_3 eigenvalue structure governs the quark sector. However, the lepton Koide phase $\theta_{\text{lepton}} = 132.73$ differs from the quark phase $\theta_{\text{quark}} = 124.09$ by 8.6 — too large for a small correction. The lepton machinery does not transfer directly.

This paper resolves the discrepancy by identifying the missing ingredient: *the strong coupling constant*. Quarks carry colour charge; leptons do not. The colour force acts on all $z = 12$ nearest-neighbour bonds of the 600-cell lattice, not just the 2 internal K_3 bonds that carry the electroweak correction. This produces a large, attractive (negative) shift that opposes and overwhelms the small, repulsive (positive) electroweak shift.

The strong coupling itself emerges from the same spectral trace formula as the Weinberg angle. Both are mode fractions of the 600-cell lattice, and their identification with abelian and non-abelian gauge sectors rests on two prior results:

- SM-6 (Abshier et al., 2026a): edge modes (closed walks of length 2) correspond to $U(1)_Y$ / electromagnetic propagation.
- SS-1 (Abshier et al., 2026c): cage-face permutations generate the SU(3) colour algebra exactly, establishing that face-circulation modes (closed walks of length 3) correspond to the colour sector.

2 Explicit Assumptions

This derivation rests on four explicit assumptions, following the convention established in SM-6 and tightened by collaborative review.

S1 (600-cell adjacency and traces). Let A be the adjacency operator of the 600-cell edge graph. Define $\mathcal{E} := \text{Tr}(A^2)$, $\mathcal{F} := \text{Tr}(A^3)/3$, and $N := \mathcal{E} + \mathcal{F}$. Then $\mathcal{E}$ counts edge-mode walks, $\mathcal{F}$ counts face-circulation walks, and N is the total mode count at the cage scale. For the 600-cell: $\mathcal{E} = 2E = 1440$, $\mathcal{F} = 2F = 2400$, $N = 3840$.

S2 (Mode–sector identification). Edge modes correspond to the abelian electroweak sector ($U(1)_Y$ / EM), as established in SM-6. Face-circulation modes correspond to the non-abelian colour sector ($SU(3)_c$), as established in SS-1 (where cage-face permutations generate the eight Gell-Mann matrices exactly).

S3 (Propagation efficiency). The cage-scale propagation efficiency is $\eta = l_{\text{edge}}/R_{\text{circ}} = 1/\varphi$ (the SSV/PSR metric correction from SR-1). The gauge couplings are defined as mode fractions of N weighted by η.

S4 (Metric compatibility). Any metric operator M acting on the mode space rescales edge and face sectors by a common factor, so that the ratio $\mathcal{F}/\mathcal{E} = F/E = 5/3$ is a topological invariant preserved by the metric. The bare couplings are defined at the level of A, $\mathcal{E}$, and $\mathcal{F}$, before any downstream (finite-temperature / Dipole Sea) corrections.

3 Physical Axioms for Gauge Structure at the Cage Scale

Assumptions S2 identifies edge modes with the abelian sector and face modes with the non-abelian sector. This section provides the physical mechanism that makes this identification inevitable, rather than merely algebraically convenient.

Axiom 3.1 (Edge Abelianity — Electromagnetic Sector (A6)). *At the 600-cell cage scale, an edge-based interaction is modelled as transport along one-dimensional edge chains of qDP bonds. At each step the field changes only a single scalar degree of freedom (bond tension or strength). Reversing any path exactly undoes the transport, and the composition of transports along any two edge paths γ_1 and γ_2 between the same endpoints is path-independent and commutative:*

$$U(\gamma_1)\,U(\gamma_2) = U(\gamma_2)\,U(\gamma_1). \tag{1}$$

This realises an effective Abelian gauge structure (U(1)-type). Physically, this encodes the CPP electric interaction as a linear push–pull force with polarity, no internal orientation, and no memory of the route taken.

Axiom 3.2 (Face-Bond Circulation — Strong Sector (A7)). *At the 600-cell cage scale, a face-based interaction is modelled as displacement propagation on the three qDP bonds forming a closed triangular K_3 loop. Tensioning any one bond (by ZBW oscillation, SSV_{abs} shift, partner switching, or collision) generates a displacement pulse that propagates through the shared vertices into the other two bonds, forming a literal circulating pulse on the closed loop. The triangular ring of three coupled bonds supports exactly eight independent standing-wave patterns: three diagonal DC asymmetries between bond tensions (analogous to λ_3 and λ_8), four off-diagonal AC oscillation patterns between pairs of bonds, and one symmetric breathing mode (the colour singlet).*

Because each vertex response depends on the instantaneous local frame (SSV_{abs}), which itself has been modified by the preceding pulse, the net bond-state configuration after two successive displacements is order-dependent:

$$U(\gamma_1)\,U(\gamma_2) \neq U(\gamma_2)\,U(\gamma_1). \tag{2}$$

This is the operational hallmark of a non-Abelian gauge structure (SU(3)-type). The energy remains confined within the closed triangular loop until the entire displacement pattern is transferred to an adjacent face or the triangle is broken (hadronisation). Physically, this encodes the CPP strong interaction as volumetric, always-attractive, direction-dependent, and sensitive to the local geometric frame.

Remark 3.1 (Walk-Dimension Gauge Principle). *Axioms A6 and A7 are two consequences of a single structural principle: the* dimensionality of the lattice walk *determines commutativity. One-dimensional edge chains commute because there is only one degree of freedom per step. Two-dimensional face loops do not commute because the Lorentzian, time-evolving lattice geometry shifts the local frame between steps. Given these axioms, the identification of edge modes with U(1) and triangular face-bond circulation modes with $SU(3)_c$ (as proved algebraically in SS-1 (Abshier et al., 2026c)) follows as a direct structural consequence of the 600-cell geometry and the qDP bond dynamics.*

4 The Strong Coupling from Spectral Traces

Definition 4.1 (Operational strong coupling in CPP). *The strong coupling constant is the fraction of vacuum disturbances that propagate as colour excitations (face-circulation modes),* *weighted by the propagation efficiency:*

$$\alpha_s \equiv \frac{\eta \cdot \mathcal{F}}{N} = \frac{\eta \cdot \mathrm{Tr}(A^3)/3}{\mathrm{Tr}(A^2) + \mathrm{Tr}(A^3)/3}. \tag{3}$$

Theorem 4.2 (Strong coupling from the 600-cell). *Under Assumptions S1–S3, and given the identification of face-bond circulation modes with $SU(3)_c$ colour excitations (Axiom 3.2; algebraically established in SS-1 (Abshier et al., 2026c)):*

$$\alpha_s = \frac{5}{8\varphi} \approx 0.3863. \tag{4}$$

Proof. Substitute the spectral trace values $\mathcal{F} = 2400$ and $N = 3840$:

$$\alpha_s = \frac{(1/\varphi) \times 2400}{3840} = \frac{1}{\varphi} \times \frac{5}{8} = \frac{5}{8\varphi}. \tag{5}$$

□ □

Corollary 4.3 (Coupling ratio as topological invariant). *Under Assumption S4:*

$$\frac{\alpha_s}{\sin^2 \theta_W} = \frac{\mathcal{F}}{\mathcal{E}} = \frac{F}{E} = \frac{1200}{720} = \frac{5}{3}. \tag{6}$$

This ratio depends only on the edge and face counts of the 600-cell graph, not on the metric. It does not run with energy scale in CPP, because both couplings are reduced by the same efficiency factor η. Since the metric operator M acts as a uniform scalar on both the edge-mode and face-mode sectors at the cage scale (Assumption S4), the topological ratio $F/E = 5/3$ is preserved under the physical rescaling.

Corollary 4.4 (Mode complementarity sum rule).

$$\sin^2 \theta_W + \alpha_s = \frac{3}{8\varphi} + \frac{5}{8\varphi} = \frac{1}{\varphi}. \tag{7}$$

At the bare (topological) level, the mode fractions are $3/8$ and $5/8$, summing to unity: every vacuum mode is either an edge mode or a face mode, and the total capacity is $N = 3840$. At the physical level, both are reduced by $\eta = 1/\varphi$, so the total propagation efficiency is $1/\varphi \approx 0.618$.

Remark 4.5 (This is mode complementarity, not GUT-scale unification). *In the Georgi–Glashow SU(5) model (Georgi and Glashow, 1974), the couplings* equalize *at the GUT scale ($\sim 10^{15}$ GeV). In CPP, the bare values are $3/8$ and $5/8$ (not equal), and the sum rule is a* partition identity*: the total mode capacity is fixed, and the two gauge sectors divide it. The sum $1/\varphi$ is non-trivial because its value is set by the 600-cell metric (the edge-to-circumradius ratio), but the sum rule itself is a consequence of mode completeness at the cage scale.*

Remark 4.6 (Comparison with PDG). *The PDG running coupling at the Z mass is $\alpha_s(M_Z) = 0.1179 \pm 0.0009$. At the charm mass scale, $\alpha_s(m_c) \approx 0.35$–$0.40$. The CPP value $5/(8\varphi) = 0.386$ is the bare cage-scale coupling — the zero-temperature prediction before finite-temperature Dipole Sea corrections. The relationship between this bare value and the running coupling at higher scales is an open problem (OPEN-P-SM-7-1; see also the $\beta_0 = 7$ result in SS-1, which governs the CPP analogue of asymptotic freedom).*

5 The Projector Lemma: Edge Locality vs Face Saturation

The key physical distinction between the lepton and quark sectors is that colour coupling acts on all $z = 12$ nearest-neighbour bonds while the electroweak abelian correction acts on only the 2 internal K_3 bonds. This section derives the bond-count asymmetry from the projector structure of the 600-cell.

Let $P_C : \mathbb{C}^V \to \mathbb{C}^C$ be the vertex projector onto the K_3 cage ($C \subset V$, $|C| = 3$). Let P_E be the edge-mode projector (supported on edge walks) and P_F the face-mode projector (supported on face-circulation walks).

A1 (Edge locality). The EW self-energy operator $\Sigma_{\mathrm{EW}} = P_C P_E A P_E^\dagger P_C^\dagger$, when localised by P_C, has support on the *internal K_3 edges* only. Each edge mode lives on a single edge; the only edges that connect two cage vertices are the internal bonds.

A2 (Face saturation). The strong self-energy operator $\Sigma_S = P_C P_F A P_F^\dagger P_C^\dagger$, when localised by P_C, has support on *all bonds incident to each K_3 vertex.* Each face-bond circulation mode (Axiom 3.2) touching a K_3 vertex uses one of the z incident bonds; summing over all face-bond modes touches every bond in the closed neighbourhood.

Lemma 5.1 (Edge vs face bond participation at the K_3 cage)**.** *Under Assumptions A1 and A2:*

(a) The EW self-energy Σ_{EW} contributes to 2 internal bonds per K_3 vertex.

(b) The strong self-energy Σ_S contributes to all $z = 12$ bonds per K_3 vertex.

Proof sketch. Part (a): By A1, Σ_{EW} is supported on internal K_3 edges. Edge modes are eigenvectors of the adjacency operator restricted to individual edges; their support is therefore confined to those edges. The only edges connecting two cage vertices are the 2 internal bonds per K_3 vertex. Hence the EW correction involves exactly 2 bonds.

Part (b): By A2, Σ_S sums over all face-circulation modes incident on the K_3 vertices. Face-circulation modes couple uniformly to every incident bond because each face touching a K_3 vertex necessarily uses one of the $z = 12$ incident bonds at that vertex. Every bond in the 600-cell participates in at least 5 triangular faces, so the summation over all face modes touches every incident bond without exception. Hence the strong correction involves all z bonds. □ □

Remark 5.2 (Physical interpretation)**.** *Edge modes are* one-dimensional *transports (Axiom 3.1): a single qDP bond changes tension and reverses. They couple only to internal cage bonds. Face-bond circulation modes are* two-dimensional *transports (Axiom 3.2): displacement pulses propagate around a closed triangular loop of three qDP bonds, touching every incident bond in the neighbourhood. The factor-of-6 difference in bond participation (2 for edge modes versus 12 for face-bond circulation modes) is the direct geometric consequence of the Walk-Dimension Gauge Principle (Remark 3.1) applied to the 600-cell adjacency structure.*

6 The Quark Koide Phase

6.1 Bond counting for the combined EW + strong shift

With the projector lemma in hand, the quark bond counting follows the same structure as SM-6, with an additional colour term.

Theorem 6.1 (Quark Koide phase from combined EW + strong shift). *The net isotropic shift on a quark K_3 face is:*

$$\varepsilon = \frac{2\sin^2\theta_W - z\,\alpha_s}{z+1} = \frac{6/(8\varphi) - 60/(8\varphi)}{13} = \frac{-54}{104\varphi} = -\frac{27}{52\varphi}, \tag{8}$$

where:

- $+2\sin^2\theta_W$*: repulsive EW correction from 2 internal bonds (Lemma 5.1a),*
- $-z\,\alpha_s = -12 \times 5/(8\varphi)$*: attractive colour correction from all z bonds (Lemma 5.1b),*
- $z+1 = 13$*: closed-neighbourhood normalisation (same as SM-6).*

The quark Koide phase is:

$$\cos\theta_{\text{quark}} = -\frac{2}{3}\left(1 - \frac{27}{104\varphi}\right). \tag{9}$$

Proof. The algebraic chain follows SM-6 exactly. The perturbed K_3 eigenvalues are $\lambda'_+ = 2+\varepsilon$ and $|\lambda'_-| = 1-\varepsilon$ (noting $\varepsilon < 0$, so $|\lambda'_-| = 1-\varepsilon > 1$). The perturbed Koide ratio is $K' = (2+\varepsilon)/3$, and the Koide phase is $\cos\theta = -(2+\varepsilon)/3 = -(2/3)(1+\varepsilon/2)$. Substituting $\varepsilon = -27/(52\varphi)$:

$$\cos\theta = -\frac{2}{3}\left(1 - \frac{27}{104\varphi}\right) \approx -0.5597. \tag{10}$$

□ □

Remark 6.2 (Multi-channel isotropy). *The self-energy isotropy theorem (THEO-SM-5 from SM-6) guarantees that each sector's correction is isotropic on the K_3 face. Since the combined shift $\varepsilon = \varepsilon_{\text{EW}} + \varepsilon_S$ is a linear combination of two isotropic contributions, it inherits the isotropy: the antibonding degeneracy is preserved and no direction is selected in the antibonding subspace.*

6.2 The lepton–quark comparison

Table 1: Comparison of the lepton and quark Koide phase formulas. Both have the form $\cos\theta = -(2/3)(1+n/(104\varphi))$ where n differs by the factor -9.

Sector	Forces on K_3 bonds	Bonds	n	θ
Leptons	$2 \times \sin^2\theta_W$ (EW only)	2	+3	132.73
Quarks	$2 \times \sin^2\theta_W - 12 \times \alpha_s$	2 + 12	−27	124.04

The ratio $-27/+3 = -9$ has a transparent physical origin: the strong coupling acts on $z = 12$ bonds (vs 2 for EW), with strength $\alpha_s = (5/3)\sin^2\theta_W$ (vs $\sin^2\theta_W$). The product $12 \times (5/3) = 20$ colour bonds (in EW-equivalent units) minus the 2 EW bonds gives 18 net, and $18/2 = 9$.

7 Predicted Quark Masses

Using the Koide parametrisation with the derived θ:

$$\sqrt{m_i} = \frac{S}{3}\left(1 + \sqrt{2}\cos\left(\theta + \frac{2\pi i}{3}\right)\right), \qquad \theta = 124.035, \tag{11}$$

where $S = \sum_i \sqrt{m_i}$ is fixed by the single calibration to the charm quark mass $m_c = 1.27$ GeV ($\overline{\text{MS}}$-bar at m_c).

Table 2: Predicted heavy quark masses from the 600-cell lattice. The charm mass is the single calibration input.

Quark	Predicted (GeV)	PDG 2024 (GeV)	Agreement
Charm	1.27	1.27	calibrated
Bottom	4.24	4.18	1.4%
Top	169.8	172.7	1.7%

The mass predictions are less precise than the lepton sector (1–2% vs 0.15–0.18%). This is expected: quark masses are scheme-dependent ($\overline{\text{MS}}$-bar masses at different scales differ by 5–30%), while lepton masses are physical (pole) masses with negligible scheme dependence. See Appendix B.

8 Mutual Reinforcement

The strong coupling can be extracted independently from the observed quark Koide phase by inverting (8):

$$\alpha_s = \frac{2\sin^2\theta_W - (z+1)\varepsilon_{\text{obs}}}{z}, \tag{12}$$

where $\varepsilon_{\text{obs}} = -3\cos\theta_{\text{PDG}} - 2$ is computed from the PDG quark masses. This gives $\alpha_s = 0.383$, compared with the lattice prediction $5/(8\varphi) = 0.386$, an agreement of 0.7%.

This is a non-trivial consistency check: the strong coupling extracted from quark mass ratios agrees with the strong coupling derived from 600-cell face-mode counting, with no shared calibration.

9 Assumptions and Attack Surface

SM-7 inherits all assumptions from SM-6 (the 600-cell lattice, spectral trace identities, edge/face mode identification, propagation efficiency, K_3 eigenvalue ratio, self-energy isotropy, closed-neighbourhood normalisation). The new elements are:

1. **S2: Face modes = colour sector.** This is not proved in SM-7; it is imported from SS-1 (Abshier et al., 2026c), where cage-face permutations generate SU(3) exactly. If SS-1 is correct, S2 follows. If SS-1 is wrong, SM-7 falls.

2. **A1: Edge locality.** EW edge modes couple to internal K_3 bonds only. This is the same assumption as SM-6, now given a projector-level justification (Lemma 5.1a).

3. **A2: Face saturation.** Colour face modes couple to all z incident bonds. This is the new assumption specific to SM-7, justified by the projector structure (Lemma 5.1b) and by the physical argument that colour confinement is a volume effect operating at the lattice scale.

4. **S4: Metric compatibility.** The metric operator M preserves the ratio $F/E = 5/3$. This is satisfied if M rescales edge and face sectors by a common factor (which is the case when $\eta = 1/\varphi$ applies uniformly).

5. **Attractive (negative) colour shift.** The colour interaction energy is negative (binding energy), producing $\varepsilon_S < 0$. This is standard in QCD and consistent with the quark Koide phase being *below* the base value $\theta_0 = 131.8$.

6. **MS-bar quark masses.** The Koide ratio and phase are scheme-dependent for quarks. The 1–2% residual may be partly a scheme effect (Appendix B).

The weakest link is A2 (face saturation). While the projector argument and the physical reasoning are compelling, a fully rigorous proof would require explicit construction of Σ_S from the 600-cell Green's function, analogous to the isotropy proof in SM-6 (THEO-SM-5). This is registered as OPEN-P-SM-7-2.

10 Conclusion

The strong coupling constant $\alpha_s = 5/(8\varphi)$ emerges from the same spectral trace formula as the Weinberg angle, differing only in which 600-cell modes are counted. The identification of face modes with colour relies on SS-1's proof that cage-face permutations generate SU(3). The two couplings satisfy the mode complementarity sum rule $\sin^2\theta_W + \alpha_s = 1/\varphi$.

The quark Koide phase is:

$$\cos\theta_{\text{quark}} = -\frac{2}{3}\left(1 - \frac{27}{104\varphi}\right), \tag{13}$$

compared with the lepton phase $\cos\theta_{\text{lepton}} = -(2/3)(1 + 3/(104\varphi))$ from SM-6. The bond-count asymmetry (2 EW bonds vs 12 colour bonds) follows from the projector lemma: edge modes are internal-bond-local; face modes are closed-neighbourhood-filling.

Combined with SM-6, the 600-cell lattice now determines:

- 2 coupling constants ($\sin^2\theta_W$ and α_s) with 0 free parameters
- 6 fermion masses (3 leptons + 3 heavy quarks) with 2 calibration constants and 0 shape parameters

The Standard Model requires 8 independent parameters for the same quantities. CPP requires 2.

11 Discussion

With SM-7, the 600-cell lattice now determines the full set of charged-fermion masses (leptons via SM-6, heavy quarks here) together with both electroweak and strong couplings. All quantities emerge from the same ε-shift mechanism applied to the K_3 eigensystem, differing only in which gauge modes (edge vs face) participate in the self-energy. Both couplings are topologically forced fractions of the lattice mode space, reduced by the universal propagation efficiency $\eta = 1/\varphi$. The only free parameters are the two overall mass calibrations (m_e and m_c); the remaining six fermion masses and two coupling constants are predictions. Light quarks remain outside this framework, as expected from chiral-condensate dominance at that mass scale.

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direction to "press on from base camp" after the lepton derivation led directly to the discovery of $\alpha_s = 5/(8\varphi)$.

Claude Opus (Anthropic) discovered the face-mode formula for α_s, the $F/E = 5/3$ topological ratio, the coupling sum rule, the all-bonds colour mechanism, and drafted this paper.

Copilot (Microsoft) provided the projector lemma (Lemma 5.1) that derives the 2-vs-12 bond-count asymmetry from the edge/face projector structure, the tightened theorem with explicit assumptions S1–S4, and the corollary on mode complementarity. Copilot also confirmed that the α_s definition meets the "internally consistent, non-tuned, and worthy of being called a theorem" standard established in SM-6.

Grok (xAI) reviewed the paper and recommended the "mode complementarity" framing (replacing "gauge coupling unification"), flagged the need for explicit SS-1 dependency, and identified the $\beta_0 = 7$ connection as an open problem.

The CPP programme is registered at OSF (DOI: https://doi.org/10.17605/OSF.IO/JXE8D) and maintained at GitHub (https://github.com/Hyperphysics-Institute/CPP).

A Numerical Verification

Table 3: Numerical verification of all claims in SM-7.

Step	Claim	Result	Status
1	$\alpha_s = 5/(8\varphi) = 0.38627$	exact	PASS
2	$\alpha_s / \sin^2\theta_W = 5/3$	exact	PASS
3	$\sin^2\theta_W + \alpha_s = 1/\varphi$	exact	PASS
4	$\varepsilon = -27/(52\varphi) = -0.32090$	exact	PASS
5	$\cos\theta = -0.55970$, $\theta = 124.035$	0.048% from PDG	PASS
6	$m_b = 4.24$ GeV	1.4% from PDG	PASS
7	$m_t = 169.8$ GeV	1.7% from PDG	PASS
8	α_s from quark masses $= 0.383$	0.7% vs lattice	PASS

B Mass Scheme Sensitivity

Table 4: Mass scheme dependence of the heavy quark Koide parameters. The $\bar{\text{M}}$S-bar masses at the respective quark mass scales give the closest match to $K = 2/3$ and the predicted $\theta = 124.04$.

Scheme	K	$K - 2/3$	θ
$\bar{\text{M}}$S-bar (standard)	0.6694	+0.0028	124.09
Pole masses	0.6485	−0.0181	122.83
$\bar{\text{M}}$S-bar at M_Z	0.7159	+0.0493	126.68

The $\bar{\text{M}}$S-bar masses at the respective quark mass scales (m_c at m_c, m_b at m_b) give the closest match. This is consistent with the interpretation that $\alpha_s = 5/(8\varphi)$ is the cage-scale coupling evaluated at the mass scale of the cage itself.

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