

# SM-5: Tribimaximal Neutrino Mixing as the Zeroth-Order PMNS Matrix from the K3 Cage Base Graph

600-Cell Standard Model Emergence Series

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## Abstract

The three neutrino mass eigenstates in Conscious Point Physics (CPP) are identified as the eigenmodes of the K3 ZBW Hamiltonian  $\hat{H}_{\text{ZBW}} = \hbar\omega_0 A_{K_3}$  derived in SM-3 [8]. The three charged lepton mass eigenstates are the vertex states of K3 (established in SM-4 [9]). The PMNS neutrino mixing matrix is therefore the change-of-basis matrix between these two representations — exactly the *tribimaximal mixing matrix*  $U_{\text{TBM}}$ , derived from K3 eigenstructure given this identification (ansatz, not yet derived from first principles):

$$\sin^2 \theta_{12}^{(0)} = \frac{1}{3}, \quad \sin^2 \theta_{23}^{(0)} = \frac{1}{2}, \quad \sin^2 \theta_{13}^{(0)} = 0.$$

The observed deviations from tribimaximal mixing ( $\sin^2 \theta_{12} = 0.304$ ,  $\sin^2 \theta_{23} = 0.570$ ,  $\sin^2 \theta_{13} = 0.022$ ) are second-order corrections from charged-lepton mixing and the Capotauro bias, registered as Open Problems OP-SM-4 and OP-SM-7d. Neutrino masses and the CP-violating phase  $\delta_{\text{CP}}$  are not derived here; they require the electroweak sector (SM-6, planned).

## 1 Prerequisites and Physical Picture

This paper continues directly from SM-3 (K3 Spectral Theorem [8]) and SM-4 (charged lepton masses [9]). The following are used:

- The cage base  $K_3 = \{V_1, V_2, V_3\}$  is the equilateral triangle with three colour vertices. C3 symmetry is exact (SM-1, Theorem 1 [7]).
- The ZBW Hamiltonian  $\hat{H}_{\text{ZBW}} = \hbar\omega_0 A_{K_3}$  has eigenvalues  $\lambda_+ = +2$  (bonding, once) and  $\lambda_- = -1$  (antibonding, twice). The eigenstates are:

$$|\phi_+\rangle = \frac{1}{\sqrt{3}}(1, 1, 1)^T, \tag{1}$$

$$|\phi_-^{(1)}\rangle = \frac{1}{\sqrt{6}}(2, -1, -1)^T, \tag{2}$$

$$|\phi_-^{(2)}\rangle = \frac{1}{\sqrt{2}}(0, -1, 1)^T. \tag{3}$$

- **Charged lepton mass eigenstates** (SM-4 [9]): The three lepton generations  $e, \mu, \tau$  correspond to thermal occupation of the *vertex states*  $|V_1\rangle, |V_2\rangle, |V_3\rangle$  of K3.
- **Neutrino identification** (this paper): the three neutrino species  $\nu_1, \nu_2, \nu_3$  are identified with the *eigenmode states*  $|\phi_-^{(1)}\rangle, |\phi_+\rangle, |\phi_-^{(2)}\rangle$  of  $\hat{H}_{\text{ZBW}}$  (Proposition 2.1).

## 2 The Neutrino Identification

**Proposition 2.1** (Neutrino species as K3 ZBW eigenmodes — ansatz). *We identify (ansatz, not derived) the three neutrino mass eigenstates with the three eigenmodes of  $\hat{H}_{\text{ZBW}} = \hbar\omega_0 A_{K_3}$ :*

$$\begin{aligned}\nu_1 &\leftrightarrow |\phi_-^{(1)}\rangle = \frac{1}{\sqrt{6}}(2, -1, -1)^T, \\ \nu_2 &\leftrightarrow |\phi_+\rangle = \frac{1}{\sqrt{3}}(1, 1, 1)^T, \\ \nu_3 &\leftrightarrow |\phi_-^{(2)}\rangle = \frac{1}{\sqrt{2}}(0, -1, 1)^T.\end{aligned}$$

*Physical motivation (ansatz).* Charged leptons are ZBW vertex excitations: the lepton occupies a specific colour vertex  $V_i$  (SM-3 K3 Spectral Theorem). The identification of neutrino mass eigenstates with K3 eigenmodes is the natural complementary choice (eigenmodes vs. vertex states): charged leptons couple to the cage through localised vertex occupation, while neutrinos, carrying no colour charge, propagate as global oscillation modes of the cage Hamiltonian. This complementarity is physically motivated but is not derived from CPP interaction rules. A derivation from first principles is Open Problem 1.  $\square$

**Remark 2.1** (Connection to discrete symmetry literature). *TBM was proposed by Harrison, Perkins and Scott [3] and has been derived from the discrete group  $A_4$  by Ma, Altarelli, Feruglio and others [4, 5]. The K3 adjacency matrix  $A_{K_3}$  generates the regular representation of  $\mathbb{Z}_3$ , whose Clebsch–Gordan coefficients are the TBM matrix elements. The CPP contribution is not the mathematical observation (which is known) but the physical identification: K3 is the colour-cage base of the 600-cell, not an abstract flavour symmetry postulated ad hoc.*

**Remark 2.2.** *The ordering  $\nu_1 \leftrightarrow \phi_-^{(1)}, \nu_2 \leftrightarrow \phi_+, \nu_3 \leftrightarrow \phi_-^{(2)}$  is conventional. The physical content is independent of this ordering.*

## 3 The PMNS Mixing Matrix

**Theorem 3.1** (Tribimaximal mixing from K3 eigenvectors). *The zeroth-order PMNS mixing matrix in the CPP lepton sector is exactly the tribimaximal mixing matrix:*

$$U_{\text{PMNS}}^{(0)} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \quad (4)$$

*The corresponding mixing angles are:*

$$\sin^2 \theta_{12}^{(0)} = \frac{1}{3}, \quad \sin^2 \theta_{23}^{(0)} = \frac{1}{2}, \quad \sin^2 \theta_{13}^{(0)} = 0, \quad \delta_{\text{CP}}^{(0)} = \text{undefined}. \quad (5)$$

*Proof.* The PMNS matrix element  $U_{\alpha i}$  is the amplitude for charged lepton flavour  $\alpha$  to oscillate into neutrino mass eigenstate  $\nu_i$ :

$$U_{\alpha i} = \langle V_\alpha | \phi_i \rangle, \quad (6)$$

where  $|V_\alpha\rangle \in \{|V_1\rangle, |V_2\rangle, |V_3\rangle\}$  are the lepton vertex states and  $|\phi_i\rangle$  are the K3 eigenmodes from Proposition 2.1. Computing all nine matrix elements from (1)–(3):

$$\begin{aligned} U_{e1} &= \langle V_1 | \phi_-^{(1)} \rangle = 2/\sqrt{6} = \sqrt{2/3}, \\ U_{e2} &= \langle V_1 | \phi_+ \rangle = 1/\sqrt{3}, \\ U_{e3} &= \langle V_1 | \phi_-^{(2)} \rangle = 0, \quad \dots \end{aligned}$$

The complete matrix (4) follows. Unitarity is immediate since  $\{|\phi_i\rangle\}$  is an orthonormal basis.  $\square$

**Remark 3.1** (Why TBM emerges). *The tribimaximal pattern reflects the mismatch between the two natural bases of K3:*

- The **vertex basis** is the charged-lepton basis: each lepton generation occupies a specific colour vertex.
- The **eigenmode basis** is the neutrino basis: each neutrino species is a global oscillation mode of the K3 ZBW Hamiltonian.

*These two bases are related by the K3 eigenvector matrix, which is exactly  $U_{\text{TBM}}$ . No free parameters are required given the ansatz.*

**Remark 3.2** (Relation to  $A_4$  discrete symmetry models). *The derivation of TBM from  $A_4$  [4, 5] is well established. The K3 complete graph has  $\mathbb{Z}_3 \cong A_3$  as its rotation symmetry, a subgroup of  $A_4$ . The mathematical fact that K3 eigenvectors give the TBM matrix follows.*

*The CPP-specific contribution is identifying K3 as a physical structure — the base of the 600-cell tetrahedral cage, whose  $C_3$  symmetry is derived from 600-cell geometry (SM-1, Theorem 1 [7]) rather than postulated as a discrete flavour symmetry. In CPP, the K3 symmetry is a consequence of the cage geometry; in  $A_4$  models it is an input. This grounds TBM in the same geometric structure that gives  $\delta = 1/3$  (charges) and  $K = 2/3$  (Koide ratio).*

## 4 Consistency with Observation

**Proposition 4.1** (Comparison with NuFIT data). *The tribimaximal mixing angles from Theorem 3.1 are consistent with current data (NuFIT 5.3 [2]) at zeroth order:*

Angle	TBM	NuFIT central	$1\sigma$	Status
$\sin^2 \theta_{12}$	0.333	0.304	$\pm 0.012$	$2.4\sigma$ off
$\sin^2 \theta_{23}$	0.500	0.570	$\pm 0.024$	$2.9\sigma$ off
$\sin^2 \theta_{13}$	0	0.0220	$\pm 0.0006$	$> 30\sigma$ off

*The deviations are systematic second-order corrections (Open Problem 2), not zeroth-order failures.*

**Remark 4.1** (TBM is excluded as an exact result). *TBM is not consistent with experiment as an exact statement. Daya Bay measured  $\sin^2 \theta_{13} = 0.022$  at  $> 30\sigma$ , definitively excluding  $\theta_{13} = 0$ . TBM should be read as a zeroth-order starting point. The CPP contribution is identifying the geometric origin and specifying that the required corrections come from the Capotauro mechanism and charged-lepton diagonalisation — not ad hoc fitting.*

## 5 K3 as the Common Origin

**Remark 5.1** (K3 encodes charge, mass, and mixing). *The same K3 graph gives four Standard Model results:*

<i>Paper</i>	<i>Result</i>	<i>K3 structure used</i>
<i>SM-3</i>	<i>Koide <math>K = 2/3</math></i>	<i>Spectral ratio <math>\lambda_+ /  \lambda_-  = 2</math></i>
<i>SM-4</i>	<i>Lepton mass constraint</i>	<i>Vertex occupation <math>\propto  \psi_i ^2</math></i>
<i>SM-5</i>	<i>TBM neutrino mixing</i>	<i>Eigenvector–vertex change of basis</i>
<i>SM-1</i>	<i>Charge <math>\delta = 1/3</math></i>	<i>C3 combinatorics</i>

*The cage base triangle encodes charge quantisation (combinatorial), lepton mass ratios (spectral), and neutrino mixing angles (eigenvector structure) from the same underlying geometry.*

## 6 Open Problems

**Open Problem 1** (Neutrino identification: why eigenmodes?). *Derive from CPP interaction rules why the neutrino mass eigenstates are diagonal in the K3 eigenmode basis while charged-lepton mass eigenstates are diagonal in the K3 vertex basis. This is the foundational open problem of the CPP neutrino sector.*

**Open Problem 2** (OP-SM-5: Corrections to TBM mixing angles). *Derive the corrections:  $\Delta(\sin^2 \theta_{12}) = -0.029$ ,  $\Delta(\sin^2 \theta_{23}) = +0.070$ ,  $\sin^2 \theta_{13} = 0.022$ . Leading contributions: charged-lepton mass matrix diagonalisation and the Capotauro bias  $\chi \sim \varphi^{-1}$  (OP-SM-4).*

**Open Problem 3** (OP-SM-4: Capotauro mechanism for  $\theta_{13}$ ). *The reactor angle  $\sin^2 \theta_{13} = 0.022 \approx \varphi^{-2}/1.6$ . The Capotauro mechanism proposes a ZBW phase bias  $\chi \sim \varphi^{-1}$  mixing the  $\nu_e$ - $\nu_\tau$  sector. Formal derivation of  $\chi$  from 600-cell geometry is open. The  $\varphi^{-2}$  scaling suggests the golden-ratio geometry of the 600-cell is directly involved.*

**Open Problem 4** (OP-SM-7d: CP-violating phase  $\delta_{\text{CP}}$ ). *TBM has  $\sin^2 \theta_{13} = 0$ , so  $\delta_{\text{CP}}$  is undefined at zeroth order. The observed CP violation ( $\delta_{\text{CP}} \approx 195$ ) is a next-order effect requiring the electroweak sector. Connected to the Koide phase  $\theta$  (proved in SM-4 Theorem 2): both are electroweak quantities that cannot be derived from K3 cage geometry alone.*

**Open Problem 5** (Neutrino masses:  $\Delta m_{21}^2$ ,  $\Delta m_{32}^2$ ). *Absolute neutrino masses and mass-squared splittings are not derived from the K3 spectral structure. These require the electroweak suppression mechanism ( $\sigma = 120^{-d}$ ), subject of SM-6 (planned).*

## 7 Summary

Result	Status	Source
Neutrino ID as K3 eigenmodes	<b>Ansatz</b>	Prop. 2.1
$U_{\text{PMNS}}^{(0)} = U_{\text{TBM}}$ (exact)	<b>Derived</b>	Theorem 3.1
$\sin^2 \theta_{12}^{(0)} = 1/3$	<b>Derived</b>	Exact from K3
$\sin^2 \theta_{23}^{(0)} = 1/2$	<b>Derived</b>	Exact from K3
$\sin^2 \theta_{13}^{(0)} = 0$	<b>Derived</b>	Exact from K3
Corrections to TBM ( $\sim 10\%$ )	Open	OP-SM-5
$\delta_{\text{CP}}$	Open	OP-SM-7d, EW sector
Neutrino masses	Open	SM-6 (planned)

**The central result:** the change of basis between K3 vertex states (charged leptons) and K3 eigenmode states (neutrinos) is exactly the tribimaximal PMNS mixing matrix. The geometric origin of tribimaximal mixing is the K3 spectral structure of the 600-cell cage base.

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