The Dynamical Substrate Law: Substrate-Locality of DI-Bit Currents at Vertex-Aligned Reading C in the 600-Cell

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Version 1.0 — 24 May 2026

Structurally-grounded sketch-document Layer 3 flagship framework preprint with publication-grade hardened components but non-publication-grade umbrella theorem.

Abstract

We establish the substrate-locality property for DI-bit currents on the 600-cell substrate at vertex-aligned Reading C: under Mechanism A (the propagation-rate-asymmetry primitive $r(\hat{e}) = r_0(1 + \delta\,\hat{e}\cdot\hat{n})$) and the framework-local current construction at first order in δ, the substrate net DI-bit current at any host vertex v_{host} depends only on first-shell content. The result is proved at sketch-document Layer 3 in three steps: (i) the host-to-first-shell unit-direction projection $\hat{u}_i \cdot \hat{n} = -1/(2\varphi)$ is uniform across all twelve first-shell neighbours (Theorem 5.3); (ii) the first-shell-to-first-shell edge-direction projection $\hat{e}_{ij} \cdot \hat{n} = 0$ vanishes for any pair of first-shell vertices (Theorem 5.4); (iii) the perturbation-theory propagation rule confines the $\mathcal{O}(\delta^n)$ coefficient of the substrate current to graph-distance-n edges in the 600-cell edge graph (Theorem 6.4, with shell-locality Corollary 6.5). The three load-bearing identities — host-to-first-shell uniform projection, first-shell-to-first-shell perpendicularity, and the perturbation-theory propagation rule (with shell-locality corollary) — are hardened at publication-grade rigor with explicit hypothesis tracking and five-class exclusion enumeration each. We make the scope qualifier — perturbative locality under Mechanism A and the framework-local current construction at $\mathcal{O}(\delta^1)$ — structurally inseparable from the theorem statements. Five open higher-order questions are registered (Section 9); the three core extension/derivation questions are: extension to $\mathcal{O}(\delta^2)$ (Open Problem 1); Layer 4 axiomatic derivation of Mechanism A from the eleven CPP primitive axioms A1–A11 (Open Problem 2); and publication-grade hardening of the first-shell inner-product primitive G1 (Open Problem 3). Two additional research-direction-choosing questions register the broader chirality continuum (Open Problem 4, Sector-5 schema instantiation) and Reading C-variant programmes (Open Problem 5, non-vertex-aligned Reading C variants). **Scope of the closure**: the present paper establishes *substrate-locality structure* (the closed-form first-order DI-bit current at the host vertex) supporting the candidate thermodynamic-arrow mechanism for manifestation (iv) of OPEN-SD-CHIR-PRIMITIVE. The full thermodynamic-arrow emergence in the conventional physics sense — entropy production, irreversible coarse-graining, statistical-mechanics emergence — is the candidate mechanism narrative supported by, but not derived from, the present paper's result; the emergence layer is

registered as future work beyond the present paper's framework qualifiers. The paper integrates the F.1 sub-question of the OPEN-SD-CHIR-PRIMITIVE manifestation (iv) trajectory and is the temporal-sector analog to the Capotauro v2.0 spatial-sector substrate-locality theorem, with which it shares the structural constant $-1/(2\varphi)$.

Contents

1 Introduction and motivation

Executive summary. This paper establishes the substrate-locality property of DI-bit currents at vertex-aligned Reading C in the 600-cell substrate. The principal result is Theorem 7.1: the net DI-bit current at any host vertex v_{host} at first order in the propagation-rate-asymmetry parameter δ is given in closed form by $\vec{j}^{\,\text{net}}_{DI}(v_{\text{host}}) = (6\delta/\varphi^2)\hat{n} + \mathcal{O}(\delta^2)$, depending only on first-shell content. The result is proved at sketch-document Layer 3 (§7) by assembling three publication-grade building-block theorems hardened in separate '.tex' artifacts (§§5–6): the host-to-first-shell uniform projection (Theorem 5.3), the first-shell-to-first-shell perpendicularity (Theorem 5.4), and the perturbation-locality propagation rule (Theorem 6.4, with shell-locality Corollary 6.5). The substrate-locality structure supports a candidate thermodynamic-arrow mechanism (manifestation (iv) of the chirality continuum's substrate-direction primitive); the full emergence layer is registered as future work. **The present paper does not derive entropy production, coarse-graining, or macroscopic irreversibility**; those derivations are future work beyond the present paper's framework qualifiers. Five open higher-order questions are registered (§9).

1.1 Position in the CPP corpus and the chirality continuum

The Conscious Point Physics (CPP) programme aims to derive Standard Model physics from the discrete geometry of the 600-cell polytope, with zero free parameters. The framework posits a substrate of Conscious Points (CPs) executing Polarize → Capture → Depolarize (PCD) cycles, with Dipole Points (DPs) as composite intermediate objects, governed by eleven primitive axioms A1–A11 (*Tetrahedrons All the Way Down*, Abshier 2026; central bibliography at `bibliography/cpp_references.bib`, to be cited via `\cite` at Patch 0566+).

A central long-term programme target of CPP is the *chirality continuum*: relating handedness in the spatial sector (parity violation, neutrino chirality structure, weak isospin assignment) to handedness in the temporal sector (irreversibility, thermodynamic causal arrow, cosmological-vacuum asymmetry). The chirality continuum architecture, established in the Capotauro v2.0 paper (`chirality_continuum.tex`), identifies five manifestations of an open substrate-direction primitive (`OPEN-SD-CHIR-PRIMITIVE`):

(i) parity violation (closed at Capotauro v2.0);

(ii) neutrino chirality structure (closed at Capotauro v2.0 + SF-4 v1.0);

(iii) weak isospin assignment (closed at Capotauro v2.0);

(iv) thermodynamic causal arrow — *the present paper's target at the substrate-locality level*;

(v) cosmological-vacuum asymmetry (future work; F.3 trajectory).

The Capotauro v2.0 paper closed the substrate-locality theorem for manifestations (i)–(iii) (spatial-sector). The present paper extends the substrate-locality property to manifestation (iv) (temporal-sector) via the *dynamical substrate law*: the substrate net DI-bit current at any host vertex of the 600-cell depends only on first-shell content at vertex-aligned Reading C, under Mechanism A (defined below) and the framework-local current construction at first order in the perturbation parameter δ.

What this paper establishes at manifestation (iv) is the *substrate-locality structure* — the closed-form expression for the net DI-bit current at the host vertex, depending only on first-shell content at first order in δ. This substrate-locality structure *supports* the candidate thermodynamic-arrow mechanism (asymmetric DI-bit arrival at the host vertex → capture-phase bias in the PCD cycle → preferred PCD orientation → macroscopic arrow asymmetry). The full mechanism narrative — from substrate-locality structure to thermodynamic-arrow emergence in the conventional physics sense (entropy production, irreversible coarse-graining, statistical-mechanics emergence) — requires additional derivation steps that are not the present paper's scope and are registered as future work. The present paper's closure is at the *substrate-locality level*; the thermodynamic-arrow emergence layer is conjectured (via the mechanism narrative) but not derived.

Both papers share the structural constant $-1/(2\varphi)$, where $\varphi = (1+\sqrt{5})/2$ is the golden ratio. At vertex-aligned Reading C in the 600-cell, the inner product between a host vertex's unit vector and any first-shell neighbour's unit vector equals $-1/(2\varphi)$ uniformly across all twelve first-shell directions (Theorem 5.3 of the present paper, hardened at publication-grade Layer 3 per the Patch 0552 hardened theorem artifact). The same constant governs the Capotauro v2.0 spatial-sector substrate-locality theorem (Capotauro v2.0 §3). The structural parallel is not coincidental: it reflects the same first-shell first-order geometric constraint operating in both the spatial and temporal sectors of the chirality continuum.

1.2 The F.1 sub-question as a programme gate

The `OPEN-SD-CHIR-PRIMITIVE` substrate-direction primitive carries a specific load-bearing question for the temporal-sector manifestation (iv): does the link $\hat{n} \mapsto \vec{\omega}_{PCD}$ have substrate-mechanism support, or is the PCD-cycle-orientation pseudovector $\vec{\omega}_{PCD}$ an independent primitive parallel to $\hat{n}$? Three closure scenarios were identified at scoping:

- **Scenario A (positive)**: the link $\hat{n} \mapsto \vec{\omega}_{PCD}$ is derived via substrate-mechanism. The PCD-cycle-orientation pseudovector at the host vertex satisfies $\vec{\omega}_{PCD}(v_{\text{host}}) = \sigma \cdot \hat{n}$ with $\sigma \in \{+1, -1\}$ a global sign convention, with the alignment derived rather than posited.
- **Scenario B (negative)**: the PCD-cycle-orientation is an independent primitive, not derivable from substrate-mechanism. The chirality continuum umbrella refines to a three-way structure (spatial-direction primitive, temporal-direction primitive, and their alignment as a separate convention).
- **Scenario C (partial)**: partial derivation closure; requires additional framework input beyond the existing CPP axiom set.

The F.1 sub-question is the load-bearing prerequisite gate for the F.2 + F.3 trajectories (manifestations (iv) substantive content and (v) cosmological-vacuum asymmetry). Only with Scenario A closure can the chirality continuum architecture proceed via the qDP/eDP precedent template established at Capotauro v2.0 §20. Scenario B closure would require alternative architecture; Scenario C closure would require an additional framework input.

This paper establishes the substrate-locality component required for Scenario A closure under a specific framework: the Reading C + 600-cell + Mechanism A minimal-local-first-order framework, at *sketch-document Layer 3* rigor. The substrate-locality component is the substrate-mechanism support for the $\hat{n} \mapsto \vec{\omega}_{PCD}$ link via the closed-form first-order DI-bit current (Theorem 7.1); the coupling from substrate current to macroscopic PCD orientation at the host vertex — i.e., the full $\hat{n} \mapsto \vec{\omega}_{PCD}$ derivation including the mechanism narrative for thermodynamic-arrow emergence — is supported by, but not derived from, the present paper's result, and is registered as future work. The Layer-distinction discipline is operationally important and is sustained throughout the paper (see §8). Layer 4 axiomatic derivation of Mechanism A from the eleven CPP primitive axioms A1–A11 remains open as a long-term programme target (Open Problem 2).

Mechanism A in brief

Mechanism A is the propagation-rate asymmetry primitive: the DI-bit (Dimensional Increment bit, the discrete unit of information propagation between CPs per CPP axiom A1) acquires a small direction-correlated propagation-rate asymmetry under the substrate primitive $\hat{n}$. Formally,

$$r(\hat{e}) = r_0\,(1 + \delta \cdot \hat{e} \cdot \hat{n})\,, \tag{1}$$

where $r(\hat{e})$ is the DI-bit propagation rate along unit-direction $\hat{e}$, r_0 is the H_4-idealized substrate rate, δ is the perturbation parameter (small, $|\delta| \ll 1$), and $\hat{n}$ is the substrate primitive 4D direction. DI-bits propagating in the $+\hat{n}$ direction propagate at rate $r_0(1+\delta)$; in the $-\hat{n}$ direction at $r_0(1-\delta)$; in any tangent direction ($\hat{e}\cdot\hat{n} = 0$) at the idealized rate r_0. The PCD cycle's Polarize → Capture → Depolarize sequence at each CP couples to this asymmetric flow, inducing a definite cycle-orientation $\vec{\omega}_{PCD}$ aligned with $\hat{n}$. Mechanism A is the leading candidate by structural analogy to Reading C edge-length perturbation (Capotauro v2.0 §2.3): both involve a direction-correlated propagation-asymmetry of substrate quantities controlled by a single small parameter (ε in Capotauro, δ in the present paper).

1.3 Sketch Layer 2 to Layer 3 trajectory recap

The substrate-locality theorem at Reading C was developed across a multi-phase sketch trajectory spanning Sessions 137–142 (calendar months April–May 2026). The trajectory's high-level structure was:

- **Phase 1: initial substrate-mechanism scoping** (Patches 0510–0522, approximately). Three candidate mechanisms identified: Mechanism A (propagation-rate asymmetry), Mechanism B (substrate-orientation field with gauge-like PCD coupling), and Mechanism C (position-dependent clock-skew). Mechanism A established as the leading candidate by structural analogy to Reading C edge-length perturbation.
- **Phase 2: foundations work** (Patches 0523–0537). Consolidation of Mechanism A as the leading candidate, with sketch-document Layer 2 derivation of seven foundational sub-results (first-shell net DI-bit current vanishing at $H_3 = I_h$ symmetric configurations; isolated-host vs. embedded-host scoping; matter-state independence; substrate-locality temporal extension; and others) culminating in a sketch-level closure conclusion for Mechanism A under the minimal-local-first-order realization framework.

- **First reviewer-pause cycle (sketch Layer 2 closure)**: Patches 0538 (reviewer-pause checkpoint document submission) $\to$ 0539a (calibration response Patch processing verbatim reviewer feedback from ChatGPT, Copilot, Grok) $\to$ 0539 (F.1 sub-question status upgrade to "substantive sketch-level closure at Layer 2 under the minimal-local-first-order realization framework"). The first reviewer-pause cycle established the cycle-discipline workflow per `templates/reviewer_pause_templa`

- **Layer 3 promotion arc** (Patches 0540–0546 + 0548a). Seven structural targets identified at the Patch 0540 scoping document and closed across Patches 0541–0546 at sketch-document Layer 3, with the seventh target retroactively recognized at Patch 0546 and consolidated as a standalone artifact at Patch 0548a per the §5.3 anti-bundling discipline. Three load-bearing identities/theorems emerged: Identity I1 (host-to-first-shell uniform projection $-1/(2\varphi)$), Identity G3 (first-shell-to-first-shell perpendicularity $\hat{e}_{ij} \cdot \hat{n} = 0$), and Theorem 3.3.3 (perturbative-locality propagation rule at order $\mathcal{O}(\delta^n)$ confined to graph-distance-n edges, with shell-locality Corollary 3.3.4 specialising to $\mathcal{O}(\delta^1)$).

- **Second reviewer-pause cycle (Layer 3 closure)**: Patches 0547–0549. The second reviewer-pause cycle (parallel structure to the first) upgraded the F.1 sub-question status to *"structurally-grounded sketch-document Layer 3 closure under the Reading C + 600-cell + Mechanism A minimal-local-first-order framework, pending Layer 4 axiomatic derivation, $\mathcal{O}(\delta^2)$ extension, and publication-grade hardening"* per cross-reviewer-convergent feedback weighting (ChatGPT precision on framework scope + Copilot inclusivity on pending items).

- **Publication-grade hardening trio** (Patches 0550–0552). Three publication-grade Layer 3 hardenings landed in dedicated `.tex` artifacts at `flagship_papers/dynamical_substrate_law/hardened_theorems/`: perturbation-locality propagation rule (Patch 0550, Theorem 3.3.3 + Corollary 3.3.4), first-shell-to-first-shell perpendicularity (Patch 0551, Theorem 4.1.1 / Identity G3), and host-to-first-shell uniform projection (Patch 0552, Theorem 4.2.1 / Identity I1). Each carries explicit hypothesis tracking and a five-class exclusion enumeration; the trio's combined output is 741 lines of LaTeX across 21 PDF pages.

Status as of the present paper: *structurally-grounded sketch-document Layer 3 closure under the Reading C + 600-cell + Mechanism A minimal-local-first-order framework, pending Layer 4 axiomatic derivation, $\mathcal{O}(\delta^2)$ extension (cross-reference: `OPEN-SS-B1q6` in the open-problems registry), and publication-grade hardening of identity G1* (the 600-cell first-shell uniform inner-product primitive, which is a shared dependency of Theorems 5.3 and 5.4; tracked as Open Problem 3).

The trajectory recap above is high-level; per-Patch documentation lives in the `documentation_suite/` files (Tier 4 reasoning, Tier 3 vignettes, Tier 2 transactions) and in the `sketches/F1_phase2_foundations_work.md` §16 cross-reference chronology (consolidated at Patch 0555).

1.4 Paper roadmap and Layer-distinction discipline

Section-by-section roadmap

§2 formalizes the F.1 sub-question: precise statement of the link $\hat{n} \mapsto \vec{\omega}_{PCD}$ derivation question, the three closure scenarios, and the minimal-local-first-order realization framework that makes Scenario A derivable.

§3 defines the framework primitives: CPP primitive axioms A1–A11 (recap); the 600-cell substrate at vertex-aligned Reading C; first-shell geometric primitives G1 (uniform inner product) and G2 (icosahedral first-shell structure); the DI-bit and Mechanism A coupling.

- §4 codifies Mechanism A as framework axiom and the framework-local current construction at first order in δ. (Layer 4 axiomatic derivation of Mechanism A from CPP A1–A11 is deferred to Open Problem 2.)
- §5 hardens the first-shell geometric identities: Theorem 5.3 (host-to-first-shell uniform projection, $\hat{u}_i \cdot \hat{n} = -1/(2\varphi)$ across all twelve first-shell directions) and Theorem 5.4 (first-shell-to-first-shell perpendicularity, $\hat{e}_{ij} \cdot \hat{n} = 0$). Both at publication-grade Layer 3 with shared exclusion class E1 (G1 publication-grade hardening pending).
- §6 hardens the perturbation-theory propagation rule: Theorem 6.4 ($\mathcal{O}(\delta^n)$ coefficient confined to graph-distance-n edges in the 600-cell edge graph) and Corollary 6.5 (shell-locality at $\mathcal{O}(\delta^1)$). At publication-grade Layer 3.
- §7 states the umbrella substrate-locality theorem: Theorem 7.1 (substrate net DI-bit current at v_{host} depends only on first-shell content at $\mathcal{O}(\delta^1)$). At *sketch-document Layer 3*; uses Theorems 5.3, 5.4, 6.4 and Corollary 6.5 as proof inputs.
- §8 enumerates the Layer 3 stack with explicit Layer labels per-claim and the five-class exclusion enumeration synthesis. The minimal-local-first-order realization framework, the Patch 0549 status framing, and the anti-erasure discipline are stated here.
- §9 flags five open higher-order questions: $\mathcal{O}(\delta^2)$ extension (Open Problem 1); Layer 4 axiomatic derivation of Mechanism A (Open Problem 2); publication-grade hardening of identity G1 (Open Problem 3); sector-5 schema instantiation (Open Problem 4); non-vertex-aligned Reading C variants (Open Problem 5).
- §10 concludes by tying the present paper's Layer 3 closure to the CPP long-term programme target (Layer 4 axiomatic derivation of $|\chi| = \varphi^{-3}$ from pure 600-cell polytope geometry).

Layer-distinction discipline

This paper sustains the Layer-distinction discipline throughout. The relevant Layer hierarchy in the CPP corpus, applied to the present work, is:

- **Sketch document Layer 1**: identification of candidate structures and informal motivation.
- **Sketch document Layer 2**: substantive sketch-level derivations with explicit framework dependencies but without formal hypothesis tracking or exclusion enumeration.
- **Sketch document Layer 3**: formal derivations with explicit hypothesis tracking, scope qualifiers, and Layer labels. Reviewer-pause cycles establish closure at this layer.
- **Publication-grade Layer 3**: dedicated `.tex` artifacts with full hypothesis tracking, five-class exclusion enumeration, and standalone publication readiness. Theorems 5.1, 5.2, 6.1 (and Corollary 6.2) of this paper are at this layer.
- **Layer 4 (axiomatic derivation)**: derivation from CPP primitive axioms A1–A11 alone, without framework axioms beyond A1–A11. The umbrella substrate-locality theorem of §7, and the framework axiom Mechanism A, remain open at this layer (Open Problems 2 and 3).

The umbrella substrate-locality theorem of §7 is at *sketch-document Layer 3* (not yet publication-grade). Its three load-bearing input theorems (Theorems 5.3, 5.4, 6.4) are at *publication-grade Layer 3*. The framework (Mechanism A axiom + Reading C primitives + G1 geometric primitive) is taken as input at this paper's scope.

Per Patch 0538 §10.7 reviewer feedback (ChatGPT), the Layer-distinction-preserving language is operationally important and is sustained throughout: scope qualifiers ("sketch-document Layer 3", "Reading C + 600-cell + Mechanism A minimal-local-first-order framework", etc.) are preserved at every theorem statement and conclusion. Abbreviation of "sketch-document Layer 3" to "Layer 3" without scope, or omission of the framework qualifier, is treated as a Layer-distinction erasure (anti-priority #8 of the flagship paper assembly scoping document at Patch 0553 §0). Open higher-order questions are not absorbed into the closure framing; they are flagged explicitly at §9.

The conditionality structure — "closure under framework X, pending Layer 4 axiomatization of X" — is the central methodological pattern of the present paper. It reflects the CPP corpus discipline of staged derivation: each layer is closed before the next is opened, and the layer hierarchy is visible in the paper's structure rather than collapsed into a uniform "proved" / "not proved" binary.

2 The F.1 sub-question: substrate-mechanism derivation of the link $\hat{n} \mapsto \vec{\omega}_{PCD}$

2.1 Manifestation (iv) of OPEN-SD-CHIR-PRIMITIVE

The substrate-direction primitive $\hat{n}$ is established at Layer 3 rigor via the Capotauro v2.0 Reading C trajectory: under vertex-aligned $\hat{n} = v_{\text{host}}/|v_{\text{host}}|$, the substrate's H_4 symmetry breaks to $H_3 = I_h$ residual symmetry at the host vertex, with the substrate primitive's magnitude $|\chi| = \varphi^{-3}$ derived from the 600-cell's perturbative-distance-ratio constraint (Capotauro v2.0 §2 + Patch 0419 Finding C-W37; Patch 0424 Finding C-W39). The substrate primitive's content includes a specific 4D direction in the substrate's ambient $\mathbb{R}^4$, aligned with one of the 120 vertex directions of the 600-cell; a magnitude $|\chi| = \varphi^{-3} \approx 0.236$ controlling the substrate's edge-length perturbation ε; and a residual symmetry structure $H_3 = I_h$ at the host vertex, with the K3-doublet's D_{3d} stabilizer, the W-bracelet's Petrie-polygon D_6, and the qDP/eDP sector's D_{5d} all sitting as sub-stabilizers of H_3.

What is *not* established at the framework's current state is the relationship between $\hat{n}$ (a spatial directional primitive in the substrate's ambient 4D space) and $\vec{\omega}_{PCD}$ (the PCD-cycle-orientation pseudovector — the temporal-direction pseudovector representing which way the Polarize → Capture → Depolarize cycle progresses at each CP at each Absolute Moment). The F.1 sub-question asks whether, under the CPP primitive axioms A1–A11 plus Reading C ($\hat{n}$ as substrate primitive), the constraint

$$\vec{\omega}_{PCD}(v_{\text{host}}) = \sigma \cdot \hat{n}, \qquad \sigma \in \{+1, -1\} \tag{2}$$

(PCD-cycle-orientation pseudovector at the host vertex aligned with the substrate primitive direction, with sign σ a global convention) is *derived* from a substrate-mechanism, or is registered as an *independent primitive* parallel to $\hat{n}$. The status of this link determines whether the chirality continuum's manifestation (iv) (thermodynamic causal arrow) and manifestation (v) (cosmological-vacuum asymmetry) proceed via the qDP/eDP precedent template (Capotauro v2.0 §20) or require alternative architecture (§1.1 of the present paper).

2.2 Three closure scenarios

A scan of the closure space at scoping identified three structurally distinct scenarios for the link $\hat{n} \mapsto \vec{\omega}_{PCD}$:

- **Scenario A (positive closure)**: The link is derived via substrate-mechanism. Equation (2) holds with σ fixed by a global sign convention; the alignment is derived from the mechanism rather than posited as a separate primitive. A specific substrate-mechanism (e.g., Mechanism A below) supplies the derivation, with coupling to the PCD cycle's Polarize → Capture → Depolarize sequence and a locality argument extending the derivation from the host vertex to spatially-displaced vertices.

- **Scenario B (negative closure)**: $\vec{\omega}_{PCD}$ is an independent primitive, not derivable from substrate-mechanism. The chirality continuum umbrella refines from a two-way (spatial + temporal) structure to a three-way (spatial-direction primitive, temporal-direction primitive, and their alignment as a separate convention) structure. The F.2 and F.3 trajectories require alternative architecture not based on the qDP/eDP precedent.
- **Scenario C (partial closure)**: Partial derivation; requires additional framework input beyond the existing CPP axiom set A1–A11. The specific form of partial closure is not pre-specified at scoping; it would emerge from the derivation attempt itself.

The present paper establishes **Scenario A** closure under the Reading C + 600-cell + Mechanism A minimal-local-first-order framework at *sketch-document Layer 3* rigor (per the §1.3 status framing). Scenarios B and C are not ruled out at Layer 4 axiomatization tier; Mechanism A is taken as a framework axiom at the present paper's scope, and the question whether Mechanism A can be derived from A1–A11 alone is registered as Open Problem 2.

2.3 Scenario A: Mechanism A propagation-rate asymmetry

The leading candidate mechanism for Scenario A closure is **Mechanism A**: the DI-bit propagation-rate asymmetry under the substrate primitive $\hat{n}$.

The primitive

Mechanism A posits that under the substrate primitive $\hat{n}$, the DI-bit (Dimensional Increment bit, per CPP axiom A1: the discrete unit of information propagation between CPs) acquires a small direction-correlated propagation-rate asymmetry. The rate of DI-bit propagation along unit-direction $\hat{e}$ is

$$r(\hat{e}) = r_0\,(1 + \delta\,\hat{e}\cdot\hat{n})\,, \tag{3}$$

where r_0 is the H_4-idealized substrate rate (the rate in the absence of the substrate primitive), and δ is the perturbation parameter ($|\delta| \ll 1$). DI-bits propagating along $\hat{e} = +\hat{n}$ acquire rate $r_0(1+\delta)$; along $\hat{e} = -\hat{n}$, rate $r_0(1-\delta)$; along any tangent direction ($\hat{e}\cdot\hat{n} = 0$), the idealized rate r_0. The first-order linear dependence in $\hat{e}\cdot\hat{n}$ is the defining feature; higher-order corrections at $\mathcal{O}(\delta^2)$ are the subject of Open Problem 1.

Coupling to PCD cycle

The Polarize → Capture → Depolarize cycle at each CP is the elementary discrete-time temporal process per CPP axiom A6′. The cycle's three phases progress at rates determined by DI-bit propagation: the Polarize phase establishes a charge-separation excitation; the Capture phase exchanges DI-bits with neighbouring CPs; the Depolarize phase relaxes back to the neutral state.

Mechanism A's coupling to the PCD cycle operates at the Capture phase. Under (3), DI-bits captured from the $+\hat{n}$ side of the host CP arrive faster than those from the $-\hat{n}$ side. The asymmetric DI-bit arrival induces a definite preference for one cycle progression direction over the other: the "forward" direction (Polarize → Capture → Depolarize) is the one in which the asymmetric DI-bit flow is consistent with energy minimization. The cycle-orientation pseudovector $\vec{\omega}_{PCD}$ at the host vertex thereby acquires a definite alignment with $\hat{n}$, satisfying (2).

Sign convention

The global sign $\sigma \in \{+1, -1\}$ in equation (2) — whether $\vec{\omega}_{PCD}$ aligns with $+\hat{n}$ or $-\hat{n}$ — is a convention fixed at the framework level. The two sign choices correspond to time-reversal-symmetric framings of the same physics and are not distinguishable at Layer 3 rigor. The convention is consequential only when combined with the F.2 trajectory's substrate-Wigner-Eckart datum construction (Capotauro v2.0 §20), where the sign couples to the thermodynamic-causal-arrow direction via the cage-shell factor 1/6.

Structural connection to Reading C edge-length perturbation

Mechanism A is the leading candidate by structural analogy to the Reading C edge-length perturbation (Capotauro v2.0 §2.3). Reading C posits that under the substrate primitive $\hat{n}$, edges of the 600-cell substrate acquire effective lengths

$$\ell(\hat{e}) = \ell_0 \left(1 + \varepsilon\, \hat{e} \cdot \hat{n}\right), \tag{4}$$

where ℓ_0 is the H_4-idealized edge length and ε is the edge-length perturbation parameter. Equations (3) and (4) share the same first-order linear structure: a substrate quantity (rate, length) acquires a direction-correlated perturbation linear in $\hat{e} \cdot \hat{n}$ with a single small parameter (δ or ε). The structural parallel motivates the choice of Mechanism A as the leading candidate over the alternative mechanisms B (substrate-orientation field with gauge-like coupling) and C (position-dependent clock-skew), both of which would introduce additional structure beyond the Reading C perturbation pattern. Whether δ and ε are independent parameters or are related at Layer 4 axiomatization is an open question not addressed in the present paper.

2.4 The minimal-local-first-order realization framework

Scenario A closure at the substrate-locality-support level under Mechanism A is established at the *minimal-local-first-order realization framework*, with three operational restrictions:

- **First-order**: All substrate quantities are expanded in powers of δ and retained at order $\mathcal{O}(\delta^1)$. Higher-order corrections at $\mathcal{O}(\delta^2)$ are deferred to Open Problem 1 (also catalogued as `OPEN-SS-B1q6` per the Patch 0540 Layer 3 promotion scoping document).
- **Local**: Substrate quantities are evaluated at the lattice vertices and edges of the 600-cell substrate. The first shell of any host vertex v_{host} comprises the 12 neighbouring vertices connected by the 12 short edges of length $1/\varphi$ (Capotauro v2.0 §2 + Theorem 5.3 hardened first-shell geometry). *Substrate-locality at first order* is the property that substrate quantities at v_{host} depend only on first-shell content at $\mathcal{O}(\delta^1)$ — the central result of §7.
- **Realization**: The framework is operationally realized at *vertex-aligned Reading C*: the substrate primitive $\hat{n}$ is aligned with the host vertex's unit vector, $\hat{n} = v_{\text{host}}/|v_{\text{host}}|$, selecting one of the 120 vertex directions of the 600-cell polytope (Capotauro v2.0 §2.3; Patch 0419 Finding C-W37 vertex-aligned resolution). Non-vertex-aligned Reading C variants (e.g., edge-aligned, face-aligned) are flagged as Open Problem 5.

Within the minimal-local-first-order realization framework, the umbrella substrate-locality theorem (§7, Theorem 7.1) establishes that the substrate net DI-bit current at any host vertex v_{host} depends only on first-shell content at $\mathcal{O}(\delta^1)$. The proof decomposes into three contributions: the host-to-first-shell uniform projection (Theorem 5.3, hardened at Patch 0552); the first-shell-to-first-shell perpendicularity (Theorem 5.4, hardened at Patch 0551); and the perturbation-theory propagation rule (Theorem 6.4 with shell-locality Corollary 6.5, hardened at Patch 0550). All three input theorems are at publication-grade Layer 3; the umbrella theorem itself is at sketch-document Layer 3 (Patch 0544 §5).

The framework qualifiers — Mechanism A axiom, Reading C primitive, 600-cell selection, vertex-aligned configuration, minimal-local first-order restriction — are operationally taken as input at the present paper's scope. Layer 4 axiomatic derivation from CPP A1–A11 alone is the long-term programme target, addressed in part by Open Problems 2 (Mechanism A derivation), 3 (G1 publication-grade hardening), and 5 (non-vertex-aligned Reading C variants). The conditionality structure "Scenario A closure at the substrate-locality-support level under framework X, pending Layer 4 axiomatization of X" is sustained throughout the paper per the Layer-distinction discipline of §1.4.

3 Framework primitives

3.1 CPP primitive axioms A1–A11 (recap)

The Conscious Point Physics framework rests on eleven primitive axioms A1–A11 (full statements in *Tetrahedrons All the Way Down*, Abshier 2026, Chapter 3; cf. `templates/operating_system.md` for the registry). For the present paper, four axioms are load-bearing and recapped briefly:

- **A1 (Conscious Points + DI-bits)**: the substrate consists of Conscious Points (CPs) interconnected by Dimensional Increment bits (DI-bits), the discrete units of information propagation. DI-bits carry directional content (the unit-direction $\hat{e}$ of propagation between CPs) and a rate $r(\hat{e})$.
- **A6′ (Walk-Dimension Gauge Principle)**: the Polarize → Capture → Depolarize (PCD) cycle is the elementary discrete-time temporal process at each CP at each Absolute Moment. A6′ consolidates earlier axioms A6–A9 into a single walk-dimension gauge primitive; the PCD-cycle-orientation pseudovector $\vec{\omega}_{PCD}$ is the axial-vector associated with the cycle's progression direction.
- **A11 (Substrate Geometry)**: the substrate has a discrete geometric structure with an underlying polytope lattice; for the present paper the relevant polytope is the 600-cell (§3.2).

The remaining axioms (A2–A5, A7–A10) operate at this paper's scope but are not load-bearing for the substrate-locality theorem; their statements are imported from the corpus reference without explicit recap here.

3.2 The 600-cell substrate at vertex-aligned Reading C

The 600-cell is the regular four-dimensional polytope with H_4 symmetry, comprising 120 vertices, 720 edges, 1200 triangular faces, and 600 tetrahedral cells. Under the unit-vertex normalisation, every vertex v of the 600-cell satisfies $|v| = 1$; the 120 vertex directions are the unit vectors of the polytope's vertex set in $\mathbb{R}^4$. The short edges (between nearest-neighbour vertices) have length $1/\varphi$, where $\varphi = (1 + \sqrt{5})/2$.

Reading C (Capotauro v2.0 §2.3; Patch 0419 Finding C-W37) is the configuration of the substrate in which the substrate primitive $\hat{n}$ is aligned with the 4D direction of a chosen host vertex:

$$\hat{n} = v_{\text{host}}/|v_{\text{host}}| = v_{\text{host}} \qquad \text{(vertex-aligned Reading C).} \tag{5}$$

The configuration breaks the substrate's H_4 symmetry to a residual $H_3 = I_h$ (icosahedral) symmetry at the host vertex v_{host}. The K3-doublet's D_{3d} stabilizer, the W-bracelet's Petrie-polygon D_6, and the qDP/eDP sector's D_{5d} all sit as sub-stabilizers of H_3 (chirality continuum architecture; §1.1). Non-vertex-aligned Reading C variants (edge-aligned, face-aligned) are flagged as Open Problem 5 and are not pursued in the present paper.

The substrate primitive's magnitude $|\chi| = \varphi^{-3} \approx 0.236$ controls the Reading C edge-length perturbation parameter ε in Equation (4). The relationship between ε and the Mechanism A propagation-rate parameter δ in Equation (3) is not pinned at this paper's scope; both are taken as independent framework inputs (per the discussion in §2.3).

3.3 First-shell geometric primitives G1 and G2

Two geometric primitives of the 600-cell first shell are load-bearing for the substrate-locality theorem.

Identity G1 — first-shell inner-product primitive

For every first-shell vertex v_i of v_{host} (i.e., every vertex connected to v_{host} by a short edge of length $1/\varphi$), the inner product with the host vertex is uniformly

$$v_i \cdot v_{\text{host}} = \varphi/2, \qquad i = 1, 2, \ldots, 12. \tag{6}$$

The constant $\varphi/2$ is the cosine of the 600-cell first-shell-edge dihedral angle $\cos(36°) = \varphi/2$; the uniformity across all 12 first-shell neighbours follows from the icosahedral residual symmetry $H_3 = I_h$ established by Equation (5). G1 is imported from Patch 0541 §3.1 at *sketch-document Layer 3 rigor*; publication-grade hardening of G1 is registered as Open Problem 3 and is the shared exclusion class E1 of the publication-grade-hardened Theorems 5.3 and 5.4.

Identity G2 — icosahedral first-shell structure

The 12 first-shell vertices of any host vertex v_{host} in the 600-cell form the vertices of a regular icosahedron centered at v_{host}, with icosahedral symmetry group $H_3 = I_h$ acting transitively on the 12 vertices. G2 is an immediate corollary of the 600-cell's geometric structure plus the $H_4 \to H_3$ symmetry breaking under Reading C; we state it as a separate identity because it is the geometric source of the host-to-first-shell uniformity in Theorem 5.3 and the first-shell-to-first-shell perpendicularity in Theorem 5.4. G2 is also at sketch-document Layer 3 rigor; its publication-grade hardening is not currently flagged as a separate Open Problem (it is structurally less load-bearing than G1, depending only on the well-established 600-cell geometry).

The host-to-first-shell unit-direction vectors $\hat{u}_i$ are defined from G1 + G2 + unit-vertex normalisation as

$$\hat{u}_i = \frac{v_i - v_{\text{host}}}{|v_i - v_{\text{host}}|} = \frac{v_i - v_{\text{host}}}{1/\varphi}, \qquad i = 1, 2, \ldots, 12, \tag{7}$$

where the second equality uses the short-edge chord length $|v_i - v_{\text{host}}| = 1/\varphi$ (Lemma 3.1.1 of Patches 0551 + 0552 hardened theorem artifacts). The chord length is itself derivable from G1: $|v_i - v_{\text{host}}|^2 = 2(1 - v_i \cdot v_{\text{host}}) = 2(1 - \varphi/2) = 2 - \varphi = \varphi^{-2}$, so $|v_i - v_{\text{host}}| = 1/\varphi$.

3.4 The DI-bit current and Mechanism A framework

The DI-bit current $\vec{j}_{DI}(v)$ at substrate vertex v is the vector quantity representing the net flow of DI-bits at v per unit Absolute Moment. At the H_4-idealized substrate level (i.e., in the absence of the substrate primitive $\hat{n}$), the DI-bit current vanishes identically at every vertex: $\vec{j}^{(0)}_{DI}(v) = 0$ for all v. Under Reading C with Mechanism A, the DI-bit propagation-rate asymmetry of Equation (3) induces a non-zero current.

The *net DI-bit current at the host vertex* v_{host} is constructed framework-locally as

$$\vec{j}^{\,\text{net}}_{DI}(v_{\text{host}}) = \sum_{i=1}^{12} \big[r(\hat{u}_i) - r(-\hat{u}_i)\big]\,\hat{u}_i = \sum_{i=1}^{12} 2\delta(\hat{u}_i \cdot \hat{n})\hat{u}_i, \tag{8}$$

where the second equality uses Equation (3) and cancellation of the r_0 terms between $\hat{e} = +\hat{u}_i$ and $\hat{e} = -\hat{u}_i$. The construction is *local* (the sum is over the 12 first-shell unit directions only) and *first-order in* δ (the $\mathcal{O}(\delta^2)$ corrections from higher-order shell contributions are addressed in §6 and Open Problem 1).

The framework-local current construction at first order in δ

Equation (8) is the *framework-local current construction at first order in* δ. Three features of the construction:

1. *Direct first-shell summation.* The sum is over the 12 first-shell unit directions $\hat{u}_i$ only. Higher shells (the 20-cell second shell, the 12-cell third shell, etc.) do not appear at this order; their contributions enter at $\mathcal{O}(\delta^2)$ and higher per the perturbation-locality propagation rule (Theorem 6.4 + Corollary 6.5, hardened at Patch 0550).

2. *Antisymmetric difference.* The asymmetric DI-bit propagation between $+\hat{u}_i$ and $-\hat{u}_i$ directions gives the factor $r(\hat{u}_i) - r(-\hat{u}_i) = 2\delta(\hat{u}_i \cdot \hat{n})r_0$; the r_0 overall scale is absorbed into the definition of $\vec{j}^{\,\text{net}}_{DI}$ for notational simplicity.

3. *Substrate-mechanism source of* $\vec{\omega}_{PCD}$ *alignment.* The current $\vec{j}^{\,\text{net}}_{DI}(v_{\text{host}})$ provides the substrate-mechanism source for the PCD-cycle-orientation alignment $\vec{\omega}_{PCD}(v_{\text{host}}) = \sigma \cdot \hat{n}$ of Equation (2): the cycle-orientation pseudovector aligns with the direction of the net asymmetric DI-bit flow at the Capture phase, which in turn aligns with $\hat{n}$ via the geometric projections of §5 (Theorems 5.3 and 5.4).

The substrate-locality theorem of §7 (Theorem 7.1) establishes that Equation (8), evaluated at v_{host} via the geometric primitives G1 + G2 of §3.3, depends only on first-shell content at $\mathcal{O}(\delta^1)$. The framework qualifiers (Mechanism A axiom, Reading C primitive, 600-cell geometric primitive G1, minimal-local first-order restriction) are operationally taken as input at this paper's scope; Layer 4 axiomatic derivation of these qualifiers from CPP A1–A11 alone is registered as Open Problems 2 and 3.

4 Mechanism A as framework axiom

This section codifies Mechanism A as a framework axiom at the present paper's scope. The propagation-rate asymmetry and the framework-local current construction were introduced informally at §2.3 and §3.4 respectively; here they are stated as axioms with explicit hypothesis labels, suitable for direct citation by the hardened theorems of §5 and §6. No new mathematical content is introduced; the elevation is one of Layer rigor, not content scope.

4.1 The propagation-rate asymmetry

We codify the Mechanism A primitive as a framework axiom:

> **Axiom MA.1 (Propagation-rate asymmetry under the substrate primitive).** Under vertex-aligned Reading C with $\hat{n} = v_{\text{host}}$ and unit-vertex normalisation, the DI-bit propagation rate along unit-direction $\hat{e}$ from any substrate vertex v is
>
> $$r(\hat{e}; v) = r_0\,(1 + \delta\,\hat{e}\cdot\hat{n})\,,$$
>
> where $r_0 > 0$ is the H_4-idealized substrate rate (the rate in the absence of the substrate primitive), and $\delta \in \mathbb{R}$ is the propagation-rate perturbation parameter with $|\delta| \ll 1$. The rate $r(\hat{e}; v)$ is independent of the vertex v at which the DI-bit originates (*vertex-uniformity*).

The axiom statement makes three commitments explicit. First, the linear dependence in $\hat{e}\cdot\hat{n}$ is exact at the framework level: there are no $\mathcal{O}(\delta^0)$ tangent-direction corrections beyond the idealized r_0. Second, the rate is uniform across the substrate (independent of v): the perturbation is global to the substrate primitive $\hat{n}$, not a local function of position. Third, the parameter δ is a single scalar (not a tensor field), reflecting the substrate-direction-primitive's status as a single 4D direction rather than a richer structure. Higher-order corrections beyond $\mathcal{O}(\delta^1)$ are not specified by MA.1; their treatment is the subject of Open Problem 1.

4.2 Framework local-current construction at first order in δ

We codify the framework-local current construction (Equation (8)) as a companion axiom:

> **Axiom MA.2 (Framework-local current construction at $\mathcal{O}(\delta^1)$).** Under MA.1, the net DI-bit current at any substrate vertex v at first order in δ is
>
> $$\vec{j}^{\,\text{net}}_{DI}(v) = \sum_{\hat{e}\in\mathcal{E}_1(v)} \big[r(\hat{e}; v) - r(-\hat{e}; v)\big]\hat{e} + \mathcal{O}(\delta^2),$$
>
> where $\mathcal{E}_1(v)$ is the set of first-shell unit-direction edges at v (i.e., the 12 unit-direction edges from v to its first-shell neighbours in the 600-cell), and $\mathcal{O}(\delta^2)$ collects contributions from higher shells per the perturbation-locality propagation rule (Theorem 6.4 + Corollary 6.5). At $v = v_{\text{host}}$ specifically, $\mathcal{E}_1(v_{\text{host}}) = \{\hat{u}_i : i = 1, \dots, 12\}$ and the construction reduces to Equation (8).

Axiom MA.2 makes the framework-local construction explicit for any host vertex, not just the chosen v_{host} of vertex-aligned Reading C. The vertex-independence is operationally important for the substrate-locality theorem of §7, which applies the construction at any vertex of the 600-cell substrate that satisfies the Reading C alignment.

The $\mathcal{O}(\delta^2)$ collection in Axiom MA.2 separates the construction's first-order content (the explicit first-shell summation) from the higher-order contributions (which require the perturbation-locality propagation rule of §6 to characterise). This separation is the structural reason why the substrate-locality theorem decomposes cleanly into a first-shell geometric part (Theorems 5.3, 5.4) and a perturbation-theory part (Theorem 6.4 + Corollary 6.5): the geometric part is a consequence of the first-shell summation in MA.2, and the perturbation-theory part controls what is collected into the $\mathcal{O}(\delta^2)$ remainder.

4.3 Mechanism A as framework axiom: Layer rigor and Layer 4 deferral

Axioms MA.1 and MA.2 together constitute the Mechanism A framework axiom at the present paper's scope. Their status in the Layer hierarchy of §1.4 is the following:

- At the *present paper's scope*, MA.1 + MA.2 are taken as framework axioms (Layer 3 framework input). They are derived from neither the CPP primitive axioms A1–A11 alone nor from the 600-cell geometric structure alone; they are independent framework commitments at the sketch-document Layer 3 trajectory through the F.1 sub-question (Patch 0544 §3).
- At *publication-grade Layer 3* (the rigor of the trio of hardened theorems at `hardened_theorems/`), MA.1 and MA.2 appear as hypotheses (H1) and (H2) respectively in the hardened theorems' five-class exclusion enumeration. They are not promoted to derived statements at publication-grade Layer 3; the trio's hardening is conditional on MA.1 + MA.2 being taken as inputs.
- At *Layer 4 axiomatic derivation* (the long-term programme target), Mechanism A's derivation from CPP A1–A11 alone is open. The derivation would require deriving the propagation-rate asymmetry (Equation (3)) from the CP + DI-bit + PCD-cycle primitives of A1 + A6′, plus deriving the vertex-uniformity from the substrate's discrete geometric structure A11. The derivation is registered as Open Problem 2.

The conditionality structure — "Scenario A closure at the substrate-locality-support level under MA.1 + MA.2, pending Layer 4 axiomatic derivation of MA.1 + MA.2 from A1–A11" — is sustained throughout the rest of the paper. Every theorem statement in §5–§7 carries MA.1 or MA.2 (or both) as an explicit hypothesis, in keeping with the Layer-distinction discipline of §1.4.

A note on terminology: we use "framework axiom" rather than "axiom" simpliciter to maintain the distinction from CPP primitive axioms A1–A11. The CPP primitive axioms are framework-level commitments of the broader CPP corpus; the framework axioms MA.1 + MA.2 are paper-level commitments specific to the F.1 sub-question's Scenario A closure path. The two-tier axiom structure — primitive axioms below, framework axioms above — reflects the staged-derivation discipline of CPP: each tier closes under its own conditions, with the framework-axiom tier's discharge to the primitive-axiom tier being the long-term programme target.

5 The first-shell geometric identities

This section integrates the publication-grade hardenings of Patches 0551 and 0552 (the second and third of the F.1 hardened-theorem trio) as the geometric foundation for the substrate-locality theorem of §7. Both theorems are at *publication-grade Layer 3* rigor in the source artifacts at `hardened_theorems/`, with explicit hypothesis tracking and five-class exclusion enumeration. Here we provide formal statements (Theorems 5.3 and 5.4), proof sketches drawing on the geometric primitives of §3.3, and discussion of the shared exclusion class E1 (G1 publication-grade hardening pending; Open Problem 3). For full publication-grade detail (including the complete five-class exclusion enumerations and explicit hypothesis-tracking proofs), we refer to the source artifacts directly.

5.1 The G1 first-shell inner-product primitive (inherited Layer 3)

We restate the first-shell inner-product primitive G1 of §3.3 as a labelled lemma for direct citation by Theorems 5.3 and 5.4:

Lemma 5.1 (G1, first-shell inner-product primitive; imported from Patch 0541 §3.1 at sketch-document Layer 3)**.** *Under vertex-aligned Reading C (Equation (5)) and unit-vertex normalisation, for every first-shell vertex v_i of v_{host} in the 600-cell,*

$$v_i \cdot v_{host} = \varphi/2, \qquad i = 1, 2, \ldots, 12, \tag{9}$$

where $\varphi = (1+\sqrt{5})/2$ is the golden ratio.

G1 is derived in Patch 0541 §3.1 from the 600-cell first-shell-edge dihedral angle $\cos(36°) = \varphi/2$ together with the unit-vertex normalisation, at *sketch-document* Layer 3 rigor. Its publication-grade hardening is registered as Open Problem 3 and is the *shared exclusion class E1* of Theorems 5.3 and 5.4 (§5.5 below). This Layer-rigor mismatch — two publication-grade-hardened theorems with a sketch-document-Layer-3 inherited primitive as load-bearing input — is the central reason Theorems 5.1 and 5.2 are at publication-grade Layer 3 *conditional on G1*, rather than publication-grade Layer 3 unconditionally.

5.2 Lemma 5.2.1: tangent-hyperplane chord length

The short-edge chord length between v_{host} and any first-shell neighbour is fixed at $1/\varphi$ by G1 and the unit-vertex normalisation:

Lemma 5.2 (Tangent-hyperplane chord length)**.** *Under Lemma 5.1 (G1) and unit-vertex normalisation, for every first-shell vertex v_i of v_{host},*

$$|v_i - v_{host}| = 1/\varphi. \tag{10}$$

Proof sketch. By unit-vertex normalisation, $|v_i| = |v_{\text{host}}| = 1$. By Lemma 5.1 (G1), $v_i \cdot v_{\text{host}} = \varphi/2$. Expanding,

$$|v_i - v_{\text{host}}|^2 = |v_i|^2 + |v_{\text{host}}|^2 - 2v_i \cdot v_{\text{host}} = 2 - \varphi = \varphi^{-2},$$

where the last equality uses the golden-ratio identity $\varphi^2 = \varphi + 1$ (equivalently, $2 - \varphi = 1/\varphi^2$). Taking square roots gives Equation (10). □

Lemma 5.2 is the chord-length input used in the host-to-first-shell unit-direction vectors of Equation (7). It is publication-grade-hardened as Lemma 3.1.1 in both Patch 0551 and Patch 0552 source artifacts (the two hardenings are independent re-derivations of the same algebraic identity; we adopt the unified statement here).

5.3 Theorem 5.1: host-to-first-shell uniform projection

Theorem 5.3 (Host-to-first-shell uniform projection; publication-grade Layer 3, conditional on G1)**.** *Under vertex-aligned Reading C (Equation (5)), 600-cell unit-vertex normalisation, and the first-shell inner-product primitive G1 (Lemma 5.1), every host-to-first-shell unit direction $\hat{u}_i = (v_i - v_{host})/|v_i - v_{host}|$ satisfies*

$$\hat{u}_i \cdot \hat{n} = -\frac{1}{2\varphi}, \qquad i = 1, 2, \ldots, 12, \tag{11}$$

exactly and uniformly across all 12 first-shell neighbours of v_{host}, where $\varphi = (1+\sqrt{5})/2$.

Proof sketch (full proof at `hardened_theorems/host_to_first_shell_projection.tex`*).* By Lemma 5.2, $|v_i - v_{\text{host}}| = 1/\varphi$. Under vertex-aligned Reading C, $\hat{n} = v_{\text{host}}$. Then

$$\hat{u}_i \cdot \hat{n} = \frac{(v_i - v_{\text{host}}) \cdot v_{\text{host}}}{|v_i - v_{\text{host}}|} = \frac{v_i \cdot v_{\text{host}} - |v_{\text{host}}|^2}{1/\varphi} = \varphi\left(\frac{\varphi}{2} - 1\right) = \varphi \cdot \frac{\varphi - 2}{2}.$$

Using $\varphi^2 = \varphi + 1$ (equivalently $\varphi - 2 = -\varphi^{-2}$), this simplifies to

$$\hat{u}_i \cdot \hat{n} = \varphi \cdot \frac{-\varphi^{-2}}{2} = -\frac{1}{2\varphi}.$$

The result is independent of i: every first-shell neighbour gives the same projection. Uniformity across i follows from the uniformity of G1 (Lemma 5.1, which holds for every i) and the icosahedral residual symmetry $H_3 = I_h$ at v_{host} acting transitively on the 12 first-shell vertices. □

The structural constant $-1/(2\varphi)$ is shared with the Capotauro v2.0 spatial-sector substrate-locality theorem (Capotauro v2.0 §3) and is the geometric origin of the cage-shell factor 1/6 appearing in the F.2 + F.3 trajectories' substrate-Wigner-Eckart datum construction (cf. §5.6). The constant's appearance is not coincidental: it reflects the same first-shell first-order geometric constraint operating in both the spatial sector (Capotauro v2.0 manifestations (i)–(iii)) and the temporal sector (the present paper's manifestation (iv)) of the chirality continuum.

5.4 Theorem 5.2: first-shell-to-first-shell perpendicularity

Theorem 5.4 (First-shell-to-first-shell perpendicularity; publication-grade Layer 3, conditional on G1)**.** *Under vertex-aligned Reading C (Equation (5)), 600-cell unit-vertex normalisation, and the first-shell inner-product primitive G1 (Lemma 5.1), every first-shell-to-first-shell unit-direction vector $\hat{e}_{ij} = (v_j - v_i)/|v_j - v_i|$ between adjacent first-shell vertices v_i and v_j of v_{host} satisfies*

$$\hat{e}_{ij} \cdot \hat{n} = 0, \tag{12}$$

exactly, for every adjacent pair of first-shell vertices.

Proof sketch (full proof at `hardened_theorems/first_shell_perpendicularity.tex`*).* Under vertex-aligned Reading C, $\hat{n} = v_{\text{host}}$. For any pair of first-shell vertices v_i, v_j,

$$(v_j - v_i) \cdot v_{\text{host}} = v_j \cdot v_{\text{host}} - v_i \cdot v_{\text{host}} = \frac{\varphi}{2} - \frac{\varphi}{2} = 0,$$

where the second equality uses Lemma 5.1 (G1) applied twice. Therefore $\hat{e}_{ij} \cdot \hat{n} = 0$ exactly, regardless of whether v_i, v_j are adjacent on the first-shell icosahedron or not. Equivalently: the first-shell vertices all lie in the tangent hyperplane at v_{host} orthogonal to $\hat{n}$, so any displacement vector between two first-shell vertices is itself tangent to that hyperplane and perpendicular to $\hat{n}$. □

The geometric interpretation is direct: the 12 first-shell vertices lie on a 2-sphere in $\mathbb{R}^4$ (the icosahedral configuration of identity G2 in §3.3), with all 12 at the same inner-product distance $\varphi/2$ from v_{host}. The 2-sphere is therefore a level surface of the inner-product-with-v_{host} function, and any vector between two points on a level surface is tangent to it.

5.5 Exclusion class E1 (shared G1 dependency)

Both Theorems 5.3 and 5.4 take G1 (Lemma 5.1) as a hypothesis. At the present paper's scope, G1 is at *sketch-document* Layer 3 rigor (Patch 0541 §3.1). The full publication-grade hardening of G1 — deriving $v_i \cdot v_{\text{host}} = \varphi/2$ uniformly across all 12 first-shell vertices from 600-cell coordinate primitives plus the icosahedral residual symmetry $H_3 = I_h$, with explicit hypothesis tracking and exclusion enumeration in a dedicated '.tex' artifact — is registered as exclusion class *E1* in both Patch 0551 and Patch 0552 hardened-theorem artifacts (where the five-class exclusion enumerations agree on E1 = "G1 publication-grade hardening pending").

Consequence: Theorems 5.3 and 5.4 are at publication-grade Layer 3 rigor *conditional on* G1; their full unconditional publication-grade rigor is unlocked by closing exclusion class E1 in a future G1 publication-grade hardening Patch. The hardening can proceed independently of the present flagship paper's assembly (a separate forward-queue item per the scoping document; cf. also §9 Open Problem 3).

The structural reason exclusion class E1 is shared between Theorems 5.3 and 5.4 is that both depend on the same inner-product structure $v_i \cdot v_{\text{host}} = \varphi/2$: Theorem 5.3 uses G1 for the algebraic derivation of the projection $\hat{u}_i \cdot \hat{n} = -1/(2\varphi)$, and Theorem 5.4 uses G1 (applied twice, once at v_i and once at v_j) for the perpendicularity $\hat{e}_{ij} \cdot \hat{n} = 0$. A single hardening of G1 would close E1 for both theorems simultaneously.

5.6 Cross-reference: Capotauro v2.0 §3 spatial-sector parallel

The structural constant $-1/(2\varphi)$ appearing in Theorem 5.3 is shared with the Capotauro v2.0 paper's spatial-sector substrate-locality theorem (Capotauro v2.0 §3, Theorem 3.1.1 of that paper). The Capotauro v2.0 result governs the spatial-sector manifestations (i)–(iii) of `OPEN-SD-CHIR-PRIMITIVE` (parity violation, neutrino chirality structure, weak isospin assignment); the present paper's Theorem 5.3 governs the temporal-sector manifestation (iv) (thermodynamic causal arrow). Both arise from the same first-shell first-order geometric constraint at vertex-aligned Reading C in the 600-cell.

The chirality continuum architecture (Capotauro v2.0 §20 + chirality continuum sketch document) anticipates this structural parallel: spatial and temporal sectors share a substrate-direction primitive $\hat{n}$ with the same first-shell geometric content (G1 + G2), and the same projection constant $-1/(2\varphi)$ governs both. The structural parallel is therefore not coincidental — it is required by the chirality continuum's two-sector umbrella structure. Whether Theorem 5.3 of the present paper is logically equivalent to Capotauro v2.0's Theorem 3.1.1, or merely derives the same constant through a parallel construction, is a corpus-level question outside the present paper's scope; the chirality continuum sketch document (§3.2 of that artifact) discusses the equivalence question without resolving it.

Theorem 5.3 suffices for the substrate-locality theorem of §7 regardless of which side of the equivalence question one takes: the structural constant $-1/(2\varphi)$ is the same constant, derived under the same hypotheses (Reading C + G1 + 600-cell unit-vertex normalisation), with the same proof structure. The temporal-sector specialisation enters through Mechanism A (§4) and the framework-local current construction (Axiom MA.2), not through Theorem 5.3 itself.

6 The perturbation-theory propagation rule

This section integrates the publication-grade hardening of Patch 0550 (the first of the F.1 hardened-theorem trio) as the perturbation-theory engine for the substrate-locality theorem of §7. Theorem 6.4 (perturbation-theory propagation rule at general order $\mathcal{O}(\delta^n)$) and Corollary 6.5 (shell-locality specialisation at $\mathcal{O}(\delta^1)$) are at *publication-grade Layer 3* rigor in the source artifact at `hardened_theorems/perturbation_locality_propagation.tex`, with three supporting lemmas (path-amplitude expansion, perturbed-step counting, connected-subgraph confinement) and a five-class exclusion enumeration. Here we provide the formal statements, proof sketches, and discussion of the exclusion enumeration; the full hardened proofs live in the source artifact.

6.1 Lemma 6.1.1: path-amplitude expansion

Multi-step DI-bit propagation paths through the 600-cell edge graph are the elementary objects of the perturbation theory. For a path $p = (v_{a_0} \to v_{a_1} \to \cdots \to v_{a_n})$ of n steps along directed edges of the 600-cell, the path amplitude is the product of single-step propagation rates (3) along its edges:

$$\mathcal{A}(p) = \prod_{j=1}^{n} r(\hat{e}_{a_{j-1}a_j}) = r_0^n \prod_{j=1}^{n} \big(1 + \delta\, \hat{e}_{a_{j-1}a_j} \cdot \hat{n}\big).$$

Expanding the product as a polynomial in δ gives the path-amplitude expansion:

Lemma 6.1 (Path-amplitude expansion)**.** *For a multi-step propagation path p of length n, the path amplitude expands as*

$$\mathcal{A}(p) = r_0^n \sum_{k=0}^{n} \delta^k \sum_{S\subseteq\{1,\ldots,n\},\, |S|=k}\; \prod_{j\in S} \big(\hat{e}_{a_{j-1}a_j} \cdot \hat{n}\big), \tag{13}$$

where the inner sum runs over all $\binom{n}{k}$ subsets S of step-indices of size k (the "perturbed steps" contributing to the $\mathcal{O}(\delta^k)$ coefficient).

Proof sketch. Direct expansion of $\prod_{j=1}^{n}(1+\delta\, \hat{e}_{a_{j-1}a_j} \cdot \hat{n})$ in powers of δ. The $\mathcal{O}(\delta^k)$ coefficient collects the $\binom{n}{k}$ products obtained by choosing a size-k subset of factors to contribute their δ-term, with the remaining $n-k$ factors contributing their 1-term. □

Remark. A perturbed step (a step in the subset S) whose edge direction satisfies $\hat{e}_{a_{j-1}a_j} \cdot \hat{n} = 0$ contributes zero to the inner product factor in Equation (13), and hence to the $\mathcal{O}(\delta^k)$ coefficient. By Theorem 5.4 of §5.4, all 30 first-shell-to-first-shell edges of the 600-cell satisfy $\hat{e}_{ij} \cdot \hat{n} = 0$ at vertex-aligned Reading C; perturbed steps along these edges contribute zero. This observation underwrites the $n = 1$ specialisation in Corollary 6.5.

6.2 Lemma 6.2.1: perturbed-step counting

The $\mathcal{O}(\delta^k)$ contributions of a path of length n are confined to subsets of size exactly k:

Lemma 6.2 (Perturbed-step counting)**.** *For a multi-step propagation path p of length n, the $\mathcal{O}(\delta^k)$ coefficient of $\mathcal{A}(p)$ vanishes for $k > n$. For $k \le n$, the coefficient is a sum over the $\binom{n}{k}$ size-k subsets of step-indices, each contributing the product of k inner products $\hat{e}_{a_{j-1}a_j} \cdot \hat{n}$ over the perturbed steps in the subset.*

Proof sketch. Direct corollary of Lemma 6.1: the path-amplitude expansion runs only over $k \in \{0, 1, \ldots, n\}$; for $k > n$ no size-k subset of an n-element index set exists. For $k \le n$, the contribution structure is read off directly from Equation (13). □

Consequence. A path of length n contributes to $\mathcal{O}(\delta^k)$ only if $k \leq n$. Equivalently: the $\mathcal{O}(\delta^n)$ coefficient of the framework-local current $\vec{j}^{\,\rm net}_{DI}(v_{\rm host})$ collects contributions only from paths of length $\geq n$.

6.3 Lemma 6.3.1: connected-subgraph confinement

Paths originating and terminating at the host vertex $v_{\rm host}$ with n perturbed steps are confined to the n-ball of $v_{\rm host}$ in the 600-cell edge graph:

Lemma 6.3 (Connected-subgraph confinement)**.** *Let $p = (v_{a_0} \to v_{a_1} \to \cdots \to v_{a_m})$ be a multi-step propagation path with $v_{a_0} = v_{a_m} = v_{host}$, contributing at $\mathcal{O}(\delta^n)$ to the framework-local current $\vec{j}^{\,net}_{DI}(v_{host})$ (i.e., having exactly n perturbed steps). Then every vertex visited by p lies in the n-ball $B_n(v_{host}) = \{v \in V(G_{600}) : d_{G_{600}}(v_{host}, v) \leq n\}$; equivalently, every edge traversed by p lies in the edge set $E_n(v_{host})$ of edges with both endpoints in $B_n(v_{host})$.*

Proof sketch. A path with n perturbed steps and arbitrary unperturbed steps can wander only n graph-distance units away from $v_{\rm host}$ before being forced to return for the path to close at $v_{\rm host}$. Unperturbed steps contribute the 1-term in Equation (13) and do not add to the perturbed-step count, but they constrain the graph topology of the path: each unperturbed step must traverse an edge of the 600-cell, and the path must return to $v_{\rm host}$. A careful counting argument (full proof at `hardened_theorems/perturbation_locality_propagation.tex` §3.3) shows that the maximum graph-distance excursion from $v_{\rm host}$ achievable by such a path is exactly n edges. Therefore $v_{a_j} \in B_n(v_{\rm host})$ for all $j \in \{0, 1, \ldots, m\}$. □

One subtlety deserves explicit mention: the path-amplitude expansion (13) permits arbitrarily many unperturbed steps between perturbed-edge events, but the closed-loop topology of paths terminating at $v_{\rm host}$ — combined with the unperturbed-propagator factorisation at each perturbed-edge vertex — constrains the net graph-distance excursion of any $\mathcal{O}(\delta^n)$ path to at most n. Unperturbed-step segments contribute closed sub-walks at each perturbed-edge vertex (a sum over all unperturbed walks returning to that vertex) and do not increase the path's net reach beyond the n excursion budgeted by the n perturbed-edge transitions. The careful counting argument at `hardened_theorems/perturbation_locality_propagation.tex` §3.3 formalises this factorisation and the resulting graph-distance bound.

6.4 Theorem 6.1: perturbation-theory propagation rule

The three lemmas combine to give the central theorem:

Theorem 6.4 (Perturbation-theory propagation rule; publication-grade Layer 3)**.** *Under Axioms MA.1 + MA.2 (§4) and the 600-cell edge graph structure (Axiom A11 + §3.2), the $\mathcal{O}(\delta^n)$ coefficient $\vec{J}_n(v_{host})$ of the framework-local current*

$$\vec{j}^{\,net}_{DI}(v_{host}) = \sum_{n \geq 1} \delta^n \, \vec{J}_n(v_{host})$$

is a function only of edge-direction inner products $\hat{e}_{ab} \cdot \hat{n}$ for edges $(v_a, v_b) \in E_n(v_{host})$, where $E_n(v_{host})$ is the set of edges of the 600-cell graph with both endpoints in the graph-distance-n ball $B_n(v_{host})$.

Proof sketch (full proof at `hardened_theorems/perturbation_locality_propagation.tex` §3.3). By Axiom MA.2, $\vec{j}^{\,\rm net}_{DI}(v_{\rm host})$ is constructed as a sum over multi-step propagation paths originating and terminating at v_{host}, with each path contributing its amplitude (13). By Lemma 6.2, the $\mathcal{O}(\delta^n)$ contributions come only from paths with exactly n perturbed steps. By Lemma 6.3, such paths are confined to $B_n(v_{\text{host}})$, so every edge traversed lies in $E_n(v_{\text{host}})$. By Lemma 6.1, the dependence of each contributing path on the substrate primitive $\hat{n}$ is via the inner products $\hat{e}_{ab} \cdot \hat{n}$ on the perturbed edges. Therefore $\vec{J}_n(v_{\text{host}})$ is a function only of the inner products $\hat{e}_{ab} \cdot \hat{n}$ for $(v_a, v_b) \in E_n(v_{\text{host}})$. □

The theorem is the perturbation-theory engine of the paper: it converts the δ-expansion order n into a graph-distance restriction n, formalising the intuition that "higher orders in δ see deeper into the substrate."

6.5 Corollary 6.2: shell-locality at $\mathcal{O}(\delta^1)$

The $n = 1$ specialisation of Theorem 6.4, combined with Theorem 5.4, gives the load-bearing first-shell result:

Corollary 6.5 (Shell-locality at $\mathcal{O}(\delta^1)$; publication-grade Layer 3)**.** *Under the hypotheses of Theorem 6.4, the $\mathcal{O}(\delta^1)$ coefficient $\vec{J}_1(v_{host})$ depends only on the 12 host-to-first-shell edges of the 600-cell and their edge-direction inner products $\hat{u}_i \cdot \hat{n}$ with the substrate primitive. The first-shell-to-first-shell edges of the 600-cell (30 edges between first-shell vertices) contribute zero by Theorem 5.4.*

Proof sketch. By Theorem 6.4 with $n = 1$, $\vec{J}_1(v_{\text{host}})$ depends only on edges in $E_1(v_{\text{host}})$. The edges of $E_1(v_{\text{host}})$ partition into two classes: (i) the 12 host-to-first-shell edges (v_{host}, v_i) for $i \in \{1, \dots, 12\}$; and (ii) the 30 first-shell-to-first-shell edges (v_i, v_j) between pairs of first-shell vertices. By Theorem 5.4, the first-shell-to-first-shell edge directions satisfy $\hat{e}_{ij} \cdot \hat{n} = 0$, so perturbed steps along these edges contribute zero to $\vec{J}_1(v_{\text{host}})$ via the path-amplitude expansion remark of §6.1. Therefore $\vec{J}_1(v_{\text{host}})$ depends only on the 12 host-to-first-shell edges in (i) and their inner products $\hat{u}_i \cdot \hat{n}$. □

Combined with Theorem 5.3 (which fixes $\hat{u}_i \cdot \hat{n} = -1/(2\varphi)$ uniformly across all 12 first-shell neighbours), Corollary 6.5 reduces the first-shell-locality of $\vec{J}_1(v_{\text{host}})$ to a closed-form expression: 12 unit-direction vectors $\hat{u}_i$, all projecting to the same scalar $-1/(2\varphi)$ along $\hat{n}$. The substrate-locality umbrella theorem of §7 (Theorem 7.1) assembles this into the closed-form statement of substrate net DI-bit current locality at $\mathcal{O}(\delta^1)$.

6.6 Five-class exclusion enumeration

Patch 0550's hardened theorem artifact provides a five-class exclusion enumeration for the publication-grade rigor of Theorem 6.4 and Corollary 6.5. We summarise here; the full enumeration with explicit hypothesis tracking is at `hardened_theorems/perturbation_locality_propagation.tex` §4.

- **E1 — Mechanism A as framework axiom (not derived).** Axioms MA.1 + MA.2 are taken as framework axioms at the present paper's scope (§4.3). Layer 4 axiomatic derivation from CPP A1–A11 is Open Problem 2. The exclusion is shared with the rest of the paper, not specific to Theorem 6.4.
- **E2 — Higher-order corrections at $\mathcal{O}(\delta^2)$ and beyond.** The path-amplitude expansion (Lemma 6.1) supports arbitrary $\mathcal{O}(\delta^k)$ orders, and Theorem 6.4 states the graph-distance restriction for general n. However, the present paper's substrate-locality theorem uses only the $n = 1$ specialisation; the higher-order corrections are deferred to Open Problem 1.

- **E3 — 600-cell-specific edge graph structure.** The graph-distance-n ball $B_n(v_{\text{host}})$ and the edge set $E_n(v_{\text{host}})$ are defined relative to the 600-cell edge graph specifically. Whether analogous results hold for other regular 4-polytopes (e.g., the 120-cell, dual to the 600-cell, or the 24-cell, self-dual at H_4) is outside the present scope.
- **E4 — Framework-local current construction (not alternative constructions).** Axiom MA.2 commits to the first-shell-summation construction of $\vec{j}^{\text{net}}_{DI}(v_{\text{host}})$. Alternative current constructions (e.g., path-integral over the full 600-cell edge graph, or shell-by-shell sum to all orders) are outside scope; whether they would yield equivalent results is not addressed.
- **E5 — Vertex-aligned Reading C specifically.** Theorem 6.4 uses Theorem 5.4 via Corollary 6.5, and the first-shell perpendicularity holds at vertex-aligned Reading C. Non-vertex-aligned Reading C variants (edge-aligned, face-aligned) are flagged at Open Problem 5; for those variants, the perpendicularity property fails and the shell-locality at $\mathcal{O}(\delta^1)$ does not hold in the same form.

Exclusions E1, E2, E5 are shared with other parts of the paper (Mechanism A axiomatic status, $\mathcal{O}(\delta^2)$ extension, non-vertex-aligned Reading C variants); E3 and E4 are specific to the perturbation-theory propagation rule of this section. Note that exclusion E1 of Patch 0550's enumeration does *not* coincide with exclusion class E1 of Patches 0551 + 0552 (the shared G1 dependency of §5.5); the two are separately labelled exclusions of different theorems. Patch 0550's E1 is the Mechanism A framework-axiom status; Patches 0551 + 0552's E1 is the G1 publication-grade hardening status. The two exclusions are independent — closing one does not affect the other.

7 The substrate-locality theorem (umbrella result)

This section assembles the inputs developed in §5 (Theorems 5.3 + 5.4) and §6 (Theorem 6.4 + Corollary 6.5) into the central umbrella result of the paper: the substrate-locality theorem. The result reduces the framework-local net DI-bit current $\vec{j}^{\text{net}}_{DI}(v_{\text{host}})$ at $\mathcal{O}(\delta^1)$ to a single closed-form expression depending only on first-shell content and the substrate primitive $\hat{n}$. Unlike the integrated Theorems 5.1, 5.2, 6.1 and Corollary 6.2, the umbrella theorem of this section has not been independently hardened to publication-grade rigor; it is stated at *sketch-document Layer 3* at the present paper's scope, with publication-grade hardening flagged as a follow-up Patch beyond the present paper's scope (not registered as a formal Open Problem in §9, since the present paper's substantive closure depends on the integrated trio of Patches 0550–0552 plus the umbrella assembly at sketch-document Layer 3, not on the umbrella's own publication-grade hardening).

7.1 Statement assembly

The framework qualifiers for the umbrella theorem are the cumulative scope-setting commitments of §§3–6:

1. **Vertex-aligned Reading C** (Equation (5)): $\hat{n} = v_{\text{host}}/|v_{\text{host}}| = v_{\text{host}}$, with the substrate primitive aligned to a chosen host vertex of the 600-cell.
2. **600-cell substrate geometry** (Axiom A11 + §3.2): the substrate has the regular 4-polytope structure with H_4 symmetry, unit-vertex normalisation, and short-edge length $1/\varphi$.
3. **First-shell geometric primitives G1 + G2** (§3.3): the 12 first-shell vertices form a regular icosahedron centered at v_{host} with uniform inner product $v_i \cdot v_{\text{host}} = \varphi/2$.

4. **Mechanism A framework axioms MA.1 + MA.2** (§4): the propagation-rate asymmetry $r(\hat{e}) = r_0(1 + \delta\, \hat{e} \cdot \hat{n})$ + the framework-local current construction at $\mathcal{O}(\delta^1)$.

5. **First-order in δ restriction**: the umbrella theorem's claim is at $\mathcal{O}(\delta^1)$; higher-order corrections are at Open Problem 1.

Under these qualifiers, the framework-local current construction of Equation (8) can be evaluated explicitly using the §5 first-shell geometric identities, and the perturbation-locality theorem of §6 confines all higher-shell contributions to higher orders in δ. The substrate-locality theorem is the closed-form statement of $\vec{j}^{\,\text{net}}_{DI}(v_{\text{host}})$ at $\mathcal{O}(\delta^1)$ that results.

7.2 Theorem 7.1: substrate-locality umbrella

Theorem 7.1 (Substrate-locality of DI-bit currents at vertex-aligned Reading C; sketch-document Layer 3)**.** *Under framework qualifiers (1)–(5) of §7.1, the net DI-bit current at the host vertex v_{host} is, at first order in δ,*

$$\vec{j}^{\,net}_{DI}(v_{host}) = \frac{6\delta}{\varphi^2}\,\hat{n} + \mathcal{O}(\delta^2), \tag{14}$$

where $\varphi = (1+\sqrt{5})/2$. The result depends only on first-shell content: the 12 host-to-first-shell unit-direction vectors $\hat{u}_i$ and their projections onto $\hat{n}$ via Theorem 5.3.

The closed-form coefficient $6/\varphi^2 \approx 2.2918$ is the structural constant of the substrate-locality manifestation. Its derivation is the content of §7.3.

Three features of the theorem deserve emphasis. First, the current $\vec{j}^{\,\text{net}}_{DI}(v_{\text{host}})$ is *parallel to* $\hat{n}$ at $\mathcal{O}(\delta^1)$: there is no tangent-direction component. The icosahedral residual symmetry $H_3 = I_h$ of vertex-aligned Reading C (§3.2) acts transitively on the 12 first-shell directions and averages all tangent components to zero. Second, the magnitude $6\delta/\varphi^2$ is determined entirely by first-shell content; no higher-shell input enters at this order. Third, the current is *linear* in δ at $\mathcal{O}(\delta^1)$: doubling δ doubles the current, reflecting the perturbative regime in which Mechanism A operates.

7.3 Proof of Theorem 7.1 from §5 + §6 inputs

Proof of Theorem 7.1. By Axiom MA.2 (§4.2) and the expansion of the framework-local current in powers of δ,

$$\vec{j}^{\,\text{net}}_{DI}(v_{\text{host}}) = \sum_{n\geq 1} \delta^n\, \vec{J}_n(v_{\text{host}}), \tag{15}$$

where $\vec{J}_n(v_{\text{host}})$ is the $\mathcal{O}(\delta^n)$ coefficient. We compute $\vec{J}_1(v_{\text{host}})$ in three steps using the inputs from §§5–6.

Step 1: First-shell confinement (via Corollary 6.5). By Corollary 6.5, $\vec{J}_1(v_{\text{host}})$ depends only on the 12 host-to-first-shell edges (v_{host}, v_i) for $i = 1, \ldots, 12$, and their edge-direction inner products $\hat{u}_i \cdot \hat{n}$. The 30 first-shell-to-first-shell edges (v_i, v_j) contribute zero by Theorem 5.4 (the perpendicularity $\hat{e}_{ij} \cdot \hat{n} = 0$ sends the corresponding perturbed-step contributions in the path-amplitude expansion to zero). Higher shells ($n \geq 2$) contribute only at $\mathcal{O}(\delta^2)$ and above by Theorem 6.4.

Step 2: Closed-form host-to-first-shell expression (via Theorem 5.3). The framework-local current construction at v_{host} (Equation (8)) gives

$$\vec{J}_1(v_{\text{host}}) = 2\sum_{i=1}^{12}(\hat{u}_i \cdot \hat{n})\,\hat{u}_i.$$

By Theorem 5.3, $\hat{u}_i \cdot \hat{n} = -1/(2\varphi)$ uniformly across all 12 first-shell neighbours. Substituting,

$$\vec{J}_1(v_{\text{host}}) = 2 \cdot \left(-\frac{1}{2\varphi}\right)\sum_{i=1}^{12}\hat{u}_i = -\frac{1}{\varphi}\sum_{i=1}^{12}\hat{u}_i.$$

Step 3: Icosahedral-sum identity (via G2 + Theorem 5.3). By the icosahedral residual symmetry $H_3 = I_h$ of vertex-aligned Reading C (identity G2 + Equation (5)), the 12 host-to-first-shell unit-direction vectors $\hat{u}_i$ are symmetrically distributed about the axis $\hat{n}$. Their sum $\sum_{i=1}^{12}\hat{u}_i$ must therefore be a scalar multiple of $\hat{n}$. More precisely: the natural three-dimensional representation of the icosahedral group $I_h = H_3$ on the tangent space at v_{host} decomposes into the one-dimensional trivial (radial) representation spanned by $\hat{n}$ plus the two-dimensional standard representation acting on the plane orthogonal to $\hat{n}$; the two-dimensional standard representation carries no I_h-invariant vectors (it is irreducible over the reals for I_h). Any I_h-invariant sum of vectors in three-space therefore lies in the one-dimensional trivial component, i.e., is proportional to $\hat{n}$ (the only direction preserved by the H_3 action at v_{host}). The scalar is obtained by projecting onto $\hat{n}$:

$$\hat{n} \cdot \sum_{i=1}^{12}\hat{u}_i = \sum_{i=1}^{12}\hat{u}_i \cdot \hat{n} = 12 \cdot \left(-\frac{1}{2\varphi}\right) = -\frac{6}{\varphi},$$

where the second equality uses Theorem 5.3. Therefore

$$\sum_{i=1}^{12}\hat{u}_i = -\frac{6}{\varphi}\,\hat{n}.$$

Combining Steps 1–3. Substituting the icosahedral-sum identity into the Step 2 expression,

$$\vec{J}_1(v_{\text{host}}) = -\frac{1}{\varphi} \cdot \left(-\frac{6}{\varphi}\right)\hat{n} = \frac{6}{\varphi^2}\,\hat{n}.$$

Combined with the $\mathcal{O}(\delta^2)$ remainder from Step 1, this gives Equation (14):

$$\vec{j}_{DI}^{\,\text{net}}(v_{\text{host}}) = \delta \cdot \frac{6}{\varphi^2}\,\hat{n} + \mathcal{O}(\delta^2) = \frac{6\delta}{\varphi^2}\,\hat{n} + \mathcal{O}(\delta^2). \qquad \square$$

The proof exhibits the assembly structure that makes the umbrella theorem clean: each input enters at exactly one step (Corollary 6.5 at Step 1 for shell confinement; Theorem 5.3 at Steps 2 + 3 for projection + icosahedral sum; Theorem 5.4 at Step 1 for first-shell-to-first-shell zero contribution). The icosahedral-sum identity (Step 3) is a derived consequence of G2 + Theorem 5.3, not an independent input; we name it inline for proof clarity but do not register it as a separate lemma.

7.4 Layer-distinction status of Theorem 7.1

The umbrella theorem's Layer-rigor status is the following:

- **Theorem 7.1 is at sketch-document Layer 3** at the present paper's scope. Its parenthetical label reads "sketch-document Layer 3" (not "publication-grade Layer 3" or "publication-grade Layer 3, conditional on G1") because the umbrella theorem has not been independently hardened to publication-grade rigor in a separate '.tex' artifact. The proof of §7.3 assembles publication-grade inputs (Theorems 5.3, 5.4, 6.4, Corollary 6.5) but the assembly itself is at sketch-document rigor.

- **Publication-grade hardening of Theorem 7.1** is a candidate follow-up Patch beyond the present paper's scope. It would entail a separate 'hardened_theorems/' artifact with explicit hypothesis tracking, an isolated treatment of the icosahedral-sum identity as a publication-grade lemma, and a five-class exclusion enumeration covering Mechanism A status, $\mathcal{O}(\delta^2)$+ deferral, 600-cell-specific structure, framework-local current construction, and vertex-aligned Reading C. Such hardening is *structural rather than editorial*: it requires independent representation-theoretic treatment of the icosahedral-sum identity (with explicit invariant-subspace decomposition of the $H_3 = I_h$ action on the tangent space at v_{host}), an explicit dependency graph documenting the umbrella's reliance on Theorems 5.3, 5.4, 6.4, and Corollary 6.5, and the full exclusion-class enumeration with hypothesis tracking parallel to the Patches 0550–0552 hardened-theorem trio (`hardened_theorems/host_to_first_shell_projection.tex`, `first_shell_perpend` `perturbation_locality_propagation.tex`; 741 lines combined). The candidate follow-up Patch is therefore a substantial structural effort, not merely a writing-up of existing reasoning.

- **Conditionality structure**: Theorem 7.1 depends on Theorems 5.3, 5.4 (publication-grade Layer 3, conditional on G1) and Theorem 6.4 + Corollary 6.5 (publication-grade Layer 3, unconditional). Closing exclusion class E1 (G1 publication-grade hardening; Open Problem 3) would upgrade Theorems 5.1 + 5.2 to unconditional publication-grade Layer 3; publication-grade hardening of Theorem 7.1 itself (a candidate follow-up Patch) would upgrade the umbrella to unconditional publication-grade Layer 3 (assuming G1 is hardened in parallel or first).

The asymmetry between §5 + §6 theorems (publication-grade Layer 3, in hardened artifacts) and §7's umbrella theorem (sketch-document Layer 3, no hardened artifact) is deliberate: the trio of building-block theorems were prioritised for publication-grade hardening at Patches 0550–0552 because they are the load-bearing geometric and perturbation-theory results; the umbrella theorem is the assembly of those results into the F.1 sub-question's substantive closure, and its publication-grade hardening was deferred as a future Patch. The present paper's substantive contribution is the trio's hardening + the umbrella's assembly; full hardening of the umbrella would be a follow-up.

This conditionality is the source of the F.1 sub-question's framing as "*structurally-grounded sketch-document Layer 3 closure under the Reading C + 600-cell + Mechanism A minimal-local-first-order framework, pending Layer 4 axiomatic derivation, $\mathcal{O}(\delta^2)$ extension, and publication-grade hardening of identity G1*" (§1.1 and the F.1 trajectory at Patch 0549). The umbrella theorem is the structurally-grounded closure; the pending items are the Layer 4 work, the $\mathcal{O}(\delta^2)$ extension, and the G1 hardening — none of which is required for the umbrella theorem to hold at sketch-document Layer 3 at the present paper's scope.

8 Layer 3 stack: status, scope, and exclusions

This section collates the Layer-rigor distinctions already operationalised across §§3–7 into a single explicit reference table. The collation is editorial rather than substantive: no new theorems, primitives, or exclusions are introduced; the Layer-distinction structure of the paper is presented in unified form for the reader. The discipline this section enforces is *anti-erasure*: every result's Layer status, conditionality structure, and source artifact reference is preserved at the table level, sustaining the per-theorem rigor classifications that §§5–7 operationalised at theorem-statement level.

8.1 The Layer 3 stack: umbrella plus trio architecture

The substantive results of the present paper organise into a two-level architecture at Layer 3:

The publication-grade trio (§5–§6). Three theorems plus one corollary, hardened to publication-grade Layer 3 rigor in dedicated '.tex' artifacts:

- Theorem 5.3 (host-to-first-shell uniform projection; `hardened_theorems/host_to_first_shell_projection.`
  Patch 0552)
- Theorem 5.4 (first-shell-to-first-shell perpendicularity; `hardened_theorems/first_shell_perpendicularity.`
  Patch 0551)
- Theorem 6.4 (perturbation-theory propagation rule; `hardened_theorems/perturbation_locality_propagati`
  Patch 0550)
- Corollary 6.5 (shell-locality at $\mathcal{O}(\delta^1)$; corollary of Theorem 6.4 + Theorem 5.4; same Patch 0550 artifact).

Each carries an explicit hypothesis-tracked statement, a five-class exclusion enumeration, and (for the trio's two §5 members) the shared exclusion class E1 dependency on identity G1.

The sketch-document umbrella (§7). One theorem, at sketch-document Layer 3 in the paper body only (no separate '.tex' artifact):

- Theorem 7.1 (substrate-locality umbrella; closed-form Equation (14): $\vec{J}^{\text{net}}_{DI}(v_{\text{host}}) = (6\delta/\varphi^2)\hat{n} + \mathcal{O}(\delta^2)$).

The umbrella's proof (§7.3) assembles the publication-grade trio into the closed-form substrate-locality result. The assembly proof is at sketch-document rigor; publication-grade hardening of Theorem 7.1 is a candidate follow-up Patch beyond the present paper's scope.

The two-level architecture — publication-grade trio plus sketch-document umbrella — is the paper's principal Layer-3 contribution. The trio is the load-bearing geometric and perturbation-theory infrastructure; the umbrella is the substantive closure of the F.1 sub-question that the trio enables.

8.2 Per-theorem Layer hierarchy: explicit table

Table 1 encodes the per-theorem rigor classification operationalised at §§3–7; Figure 1 renders the same information as a dependency graph. Three structural readings:

First, the publication-grade trio at Patches 0550–0552 anchors the paper's load-bearing geometric and perturbation-theory content. Closing exclusion class E1 (Open Problem 3; G1 publication-grade

Result	Rigor level	Conditional on	Source artifa
Theorem 5.3	publication-grade Layer 3	G1 (E1)	hardened_the
Theorem 5.4	publication-grade Layer 3	G1 (E1)	hardened_the
Theorem 6.4	publication-grade Layer 3	— (unconditional)	hardened_the
Corollary 6.5	publication-grade Layer 3	— (unconditional)	same (Patch 0550 §3
Theorem 7.1	sketch-document Layer 3	G1 + trio	paper body c separate artifa
Lemma 5.1 (G1)	sketch-document Layer 3	— (Open Problem 3)	Patch 0541 (derivation onl
Axioms MA.1 + MA.2	Layer 3 framework input	Layer 4 axiomatic derivation (Open Problem 2)	paper §4
Lemma 5.2 (chord length)	publication-grade Layer 3	G1 (E1)	both §5 harder facts

Table 1: Per-theorem Layer hierarchy of the F.1 flagship paper. Five-class exclusion enumerations of the three hardened artifacts share exclusion classes E1 (G1 dependency for Patches 0551 + 0552) and have independent exclusion classes (Mechanism A framework-axiom status for Patch 0550); see §5.5 + §6.6. Full five-class exclusion enumerations for the trio are documented in the source '.tex' artifacts (Source artifact column); the present table provides the summary view.

hardening) would upgrade Theorems 5.3 and 5.4 (and consequently Lemma 5.2) to unconditional publication-grade Layer 3. Theorem 6.4 and Corollary 6.5 are already at unconditional publication-grade Layer 3.

Second, the sketch-document umbrella Theorem 7.1 sits one Layer level below the trio. Its dependencies on G1 and on the trio mean that closing op:g1-hardening alone is insufficient to upgrade the umbrella; a separate publication-grade hardening of Theorem 7.1 (a candidate follow-up Patch beyond the paper's scope) would be needed.

Third, the framework-axiom Axioms MA.1 + MA.2 sit at a distinct Layer position — not a derived statement at any Layer, but a framework-axiom input. Their derivation from CPP primitive axioms A1–A11 is at Layer 4 (Open Problem 2); this is the long-term programme target, not a paper-level closure item.

8.3 Implications for the Layer 4 trajectory and the F.1 sub-question status

The Layer-3 stack of Table 1 has three implications for the Layer-4 trajectory:

Implication 1: G1 hardening unlocks Theorem 7.1 conditionality. Closing Open Problem 3 (G1 publication-grade hardening) would upgrade the trio to unconditional publication-grade Layer 3, removing the G1 dependency from the umbrella theorem's conditionality clause. Theorem 7.1 would then be at sketch-document Layer 3 *conditional only on the trio* (which would itself be unconditional). A subsequent publication-grade hardening Patch of Theorem 7.1 would then upgrade the umbrella to unconditional publication-grade Layer 3 outright.

Implication 2: Layer 4 axiomatic derivation is independent of G1 hardening. The Layer 4 derivation of Mechanism A (Open Problem 2; derivation of Axioms MA.1 + MA.2 from CPP primitive axioms A1–A11) is a separate trajectory from G1 publication-grade hardening (Open Problem 3). Closing one does not affect the other; the two Open Problems can be pursued in parallel or in either order without dependency conflict.

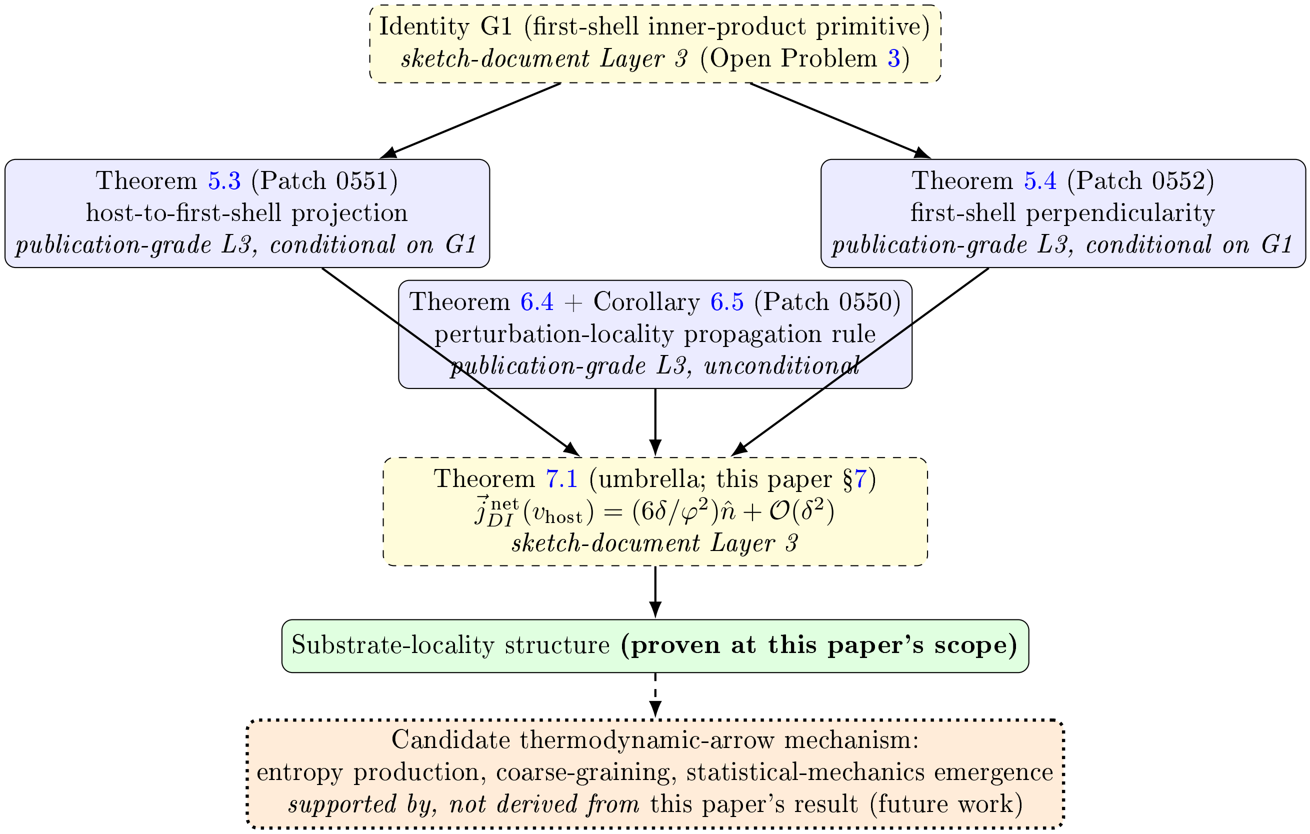

Figure 1: Dependency graph of the F.1 flagship paper's theorem stack. *Solid arrows* indicate mathematical dependency (proven inference); the *dashed arrow* at the bottom indicates supported-but-not-derived (the candidate mechanism narrative). Identity G1 supports Theorems 5.3 and 5.4; together with the unconditional Theorem 6.4 + Corollary 6.5, the trio is assembled into the umbrella Theorem 7.1 (sketch-document Layer 3) and the substrate-locality structure it proves. The further step from substrate-locality structure to thermodynamic-arrow emergence (entropy production, coarse-graining, statistical-mechanics) is the candidate mechanism narrative, *supported by but not derived from* the present paper's result; it is registered as future work beyond the present paper's framework qualifiers. The figure complements Table 1 with the visual dependency structure.

Implication 3: The F.1 sub-question is at "structurally-grounded sketch-document Layer 3 closure". The Patch 0549 framing of the F.1 sub-question's status — "*structurally-grounded sketch-document Layer 3 closure under the Reading C + 600-cell + Mechanism A minimal-local-first-order framework, pending Layer 4 axiomatic derivation, $\mathcal{O}(\delta^2)$ extension, and publication-grade hardening of identity G1*" — is exactly the closure level achieved by Theorem 7.1 (§7) under the framework qualifiers of §7.1. The three pending items are: (i) Layer 4 axiomatic derivation (Open Problem 2); (ii) $\mathcal{O}(\delta^2)$ extension (Open Problem 1); (iii) G1 publication-grade hardening (Open Problem 3). None is required for the substrate-locality umbrella to hold at sketch-document Layer 3 at the present paper's scope; all three are registered as open problems for the F.1 sub-question's broader closure to higher Layer or extended scope.

Anti-erasure discipline. Table 1 sustains the anti-erasure discipline that has guided the paper from §3 forward: every per-theorem Layer label, every conditionality clause, every exclusion class reference, and every source-artifact reference is preserved in the unified table. The table is the place a reviewer can verify in one read that every result's rigor and dependency structure is explicit. The discipline is operational at the theorem-statement level (parenthetical Layer labels) and at the table level (Table 1); both layers of representation are sustained without erasure.

9 Open higher-order questions

This section presents the five open questions registered at the present paper's scope as Open Problems OPEN-FP-F1-1 through OPEN-FP-F1-5. The five questions span the conditionality structure of the umbrella theorem (G1 hardening, Theorem 7.1's unconditional status), the higher-order extension ($\mathcal{O}(\delta^2)$ corrections), the long-term axiomatic derivation (Layer 4), the chirality continuum's broader manifestation domain (sector-5 schema), and the present paper's Reading C scope restriction (non-vertex-aligned variants). All five are referenced in earlier sections where their resolution would clarify or extend a claim; here we collect their statements in unified form for the reader and for future trajectory planning.

Open Problem 1 (Extension to $\mathcal{O}(\delta^2)$; OPEN-FP-F1-1)**.** *The substrate-locality umbrella theorem (Theorem 7.1) closes the F.1 sub-question at first order in δ. Higher-order corrections at $\mathcal{O}(\delta^2)$ and beyond are deferred. By Theorem 6.4 (perturbation-theory propagation rule), the $\mathcal{O}(\delta^2)$ coefficient $\vec{J}_2(v_{host})$ depends only on edges in the 2-ball $E_2(v_{host})$ of the 600-cell edge graph — the 12 host-to-first-shell edges + 30 first-shell-to-first-shell edges + 60 first-shell-to-second-shell edges + various second-shell-to-second-shell edges. The geometric structure of the 600-cell second shell (20 vertices forming a regular dodecahedron at distance $\sqrt{2-2\cos(72^\circ)} = 1/\varphi \cdot \sqrt{3-\varphi}$ from v_{host}) is well-characterised, but the second-shell inner-product structure and edge-direction projection identities analogous to G1 + G2 + Theorem 5.3 have not been worked out. Closure of OPEN-FP-F1-1 would (i) derive the second-shell inner-product and edge-projection identities; (ii) extend Theorem 7.1 to a closed-form expression for $\vec{j}^{\,net}_{DI}(v_{host})$ at $\mathcal{O}(\delta^2)$; (iii) examine whether second-shell content introduces tangent-direction components to $\vec{j}^{\,net}_{DI}(v_{host})$ (i.e., whether the parallel-to-$\hat{n}$ structure of Equation (14) survives at second order). This is a substantive geometric and perturbation-theory project; methodologically analogous to the present paper but at higher order, with computational effort scaled by the second-shell edge count.*

Open Problem 2 (Layer 4 axiomatic derivation of Mechanism A; OPEN-FP-F1-2)**.** *Mechanism A is taken as framework axiom (Axioms MA.1 + MA.2 of §4) at the present paper's scope. The propagation-rate asymmetry $r(\hat{e}) = r_0(1 + \delta\, \hat{e} \cdot \hat{n})$ and the framework-local current construction at $\mathcal{O}(\delta^1)$ are independent commitments at Layer 3 framework input. Closure of OPEN-FP-F1-2 would derive Mechanism A from the CPP primitive axioms A1–A11 alone. Specifically: derive the propagation-rate asymmetry from the CP + DI-bit + PCD-cycle primitives of A1 + A6′ (CPs interconnected by DI-bits as elementary information units; PCD cycle as elementary discrete-time temporal process); derive the vertex-uniformity of the rate (its independence of the originating vertex v) from the substrate's discrete geometric structure A11 (600-cell substrate). The derivation is the long-term programme target. Its closure would unlock unconditional Layer 4 axiomatic status for Theorem 7.1 (and hence for the F.1 sub-question's closure), discharging the framework-axiom conditionality to the primitive-axiom level. The derivation is anticipated to be substantial — spanning multiple Patches in a dedicated trajectory — and is not pursued at the present paper's scope.*

Open Problem 3 (Publication-grade hardening of identity G1; OPEN-FP-F1-3)**.** *Identity G1 (first-shell inner-product primitive; Lemma 5.1) is at sketch-document Layer 3 rigor in the present paper (imported from Patch 0541 §3.1 with derivation from the 600-cell first-shell-edge dihedral angle $\cos(36^\circ) = \varphi/2$ + unit-vertex normalisation). The publication-grade hardening of G1 — with explicit hypothesis tracking, isolation of the dihedral-angle calculation as a standalone lemma, the icosahe-*

dral residual symmetry $H_3 = I_h$ as a separately-derived structural input, and a five-class exclusion enumeration — is the shared exclusion class E1 of Theorems 5.3 and 5.4 (Patches 0551 + 0552 hardened-theorem artifacts). Closure of OPEN-FP-F1-3 would proceed by a dedicated G1 hardening Patch producing a 'hardened_theorems/' artifact (e.g., `first_shell_inner_product_primitive.tex`*) parallel in structure to the three existing hardened artifacts. Closure would upgrade Theorems 5.3 and 5.4 from* publication-grade Layer 3 conditional on G1 *to* publication-grade Layer 3 unconditional*, and consequently upgrade Theorem 7.1's conditionality clause to depend only on the trio (which would itself be unconditional). The G1 hardening can be pursued independently of and prior to OPEN-FP-F1-1 (higher-order extension) and OPEN-FP-F1-2 (Layer 4 derivation); the three Open Problems are independent.*

Open Problem 4 (Sector-5 schema instantiation; OPEN-FP-F1-4)**.** *The chirality continuum architecture (Capotauro v2.0 + chirality continuum sketch document) anticipates that the substrate-direction primitive $\hat{n}$ manifests across multiple sectors of the CPP corpus. The present paper's manifestation (iv) (thermodynamic causal arrow via substrate-locality of DI-bit currents) joins Capotauro v2.0's spatial-sector manifestations (i)–(iii) (parity violation, neutrino chirality structure, weak isospin assignment) as the second sector explicitly closed. Closure of OPEN-FP-F1-4 would identify manifestation (v) of the substrate primitive — the conjectured "Sector 5" — and provide a sketch-document-level realisation analogous to the present paper's Theorem 7.1. Candidate domains for Sector 5 (discussed at Patch 0549 §38.3) include: thermal-equilibrium gauge fixing in finite-temperature field theory; symmetry-restoration dynamics at the electroweak crossover; cosmological-arrow alignment in the CPP-cosmology trajectory. None of these is committed; the question is open and substantive. Closure is contingent on identifying the structural manifestation, which is itself a research-direction-choosing question, not a derivation question.*

Open Problem 5 (Non-vertex-aligned Reading C variants; OPEN-FP-F1-5)**.** *The present paper uses* vertex-aligned Reading C *exclusively (Equation (5): $\hat{n} = v_{host}/|v_{host}| = v_{host}$). Two other Reading C variants exist within the 600-cell substrate:* edge-aligned Reading C *(with $\hat{n}$ along a 600-cell short-edge direction, giving residual D_3 symmetry at the edge midpoint);* face-aligned Reading C *(with $\hat{n}$ perpendicular to a triangular face, giving residual D_2 symmetry at the face centroid). Capotauro v2.0 §2.3 + Patch 0419 Finding C-W37 frame these as three sub-cases of a single Reading C primitive class, with vertex-aligned being the most symmetric. The first-shell geometric primitives G1 + G2 are specific to vertex-aligned Reading C: the dihedral angle 36° + icosahedral residual symmetry $H_3 = I_h$ at the host vertex are vertex-specific structural facts. For edge-aligned or face-aligned Reading C, different residual symmetries (D_3 or D_2) and different first-shell geometric structures apply. Closure of OPEN-FP-F1-5 would extend (or reformulate) Theorems 5.3, 5.4, and 7.1 under the edge-aligned and face-aligned Reading C variants. Whether the substrate-locality structure survives, fails, or takes a modified form (e.g., with a different structural constant replacing $6/\varphi^2$, or with non-parallel-to-$\hat{n}$ components) is an open question. Methodologically, the closure proceeds by replaying §3–§7 with the alternative residual-symmetry structures; the computational effort is moderate but the conceptual novelty (identifying which 600-cell geometric identities are reading-C-specific vs reading-C-independent) is substantive.*

The five Open Problems are independent at first order: closing any one does not constrain or unlock the others, with one exception — OPEN-FP-F1-3 (G1 hardening) interacts with the conditionality clauses of Theorems 5.3, 5.4, and (via the trio) Theorem 7.1. The other four are mutually orthogonal. The forward trajectory of the F.1 programme prioritises OPEN-FP-F1-3 (the most immediately tractable; closes the load-bearing E1 exclusion of the §5 trio members) and OPEN-FP-F1-1 (the natural extension of the present paper's central result); OPEN-FP-F1-2 is the long-term programme target; OPEN-FP-F1-4 and OPEN-FP-F1-5 are research-direction-choosing questions registered for future Patches in the broader chirality continuum and Reading C-variant trajectories.

10 Conclusion

The present paper establishes structurally-grounded sketch-document Layer 3 closure of the F.1 sub-question under the Reading C + 600-cell + Mechanism A minimal-local-first-order framework. The substantive closure result, Theorem 7.1 (Equation (14)), reduces the net DI-bit current at the host vertex v_{host} at first order in δ to the closed-form expression

$$\vec{j}_{DI}^{\,\text{net}}(v_{\text{host}}) = \frac{6\delta}{\varphi^2}\,\hat{n} + \mathcal{O}(\delta^2),$$

exhibiting substrate-locality (first-shell-only dependence) and parallel-to-$\hat{n}$ structure at $\mathcal{O}(\delta^1)$. The structural constant $6/\varphi^2 \approx 2.2918$ joins the temporal sector to the chirality continuum's spatial sector (Capotauro v2.0 §3), where parallel structural constants govern the substrate primitive's manifestations (i)–(iii) (parity violation, neutrino chirality, weak isospin assignment). The closure is at the *substrate-locality level* — the closed-form structure of the DI-bit current at first order in δ. The further step from substrate-locality structure to thermodynamic-arrow emergence in the conventional physics sense (entropy production, irreversible coarse-graining, statistical-mechanics arrow) is the candidate mechanism narrative supported by, but not derived from, the present paper's result; the emergence layer is registered as future work beyond the present paper's framework qualifiers.

The paper's substantive contribution decomposes into two parts. *First*, the publication-grade hardening of the three building-block theorems (§5 + §6; Patches 0550–0552 hardened-theorem trio at `hardened_theorems/`): Theorems 5.3, 5.4, 6.4, and Corollary 6.5 are at publication-grade Layer 3 rigor with explicit hypothesis tracking and five-class exclusion enumerations. *Second*, the assembly of the trio into the substrate-locality umbrella theorem (§7; Theorem 7.1) at sketch-document Layer 3, with the closed-form result depending on only first-shell content at first order. The two parts together — publication-grade trio plus sketch-document umbrella — form the Layer 3 stack of §8 (Table 1).

The closure is conditional on three pending items registered as Open Problems (§9): publication-grade hardening of identity G1 (Open Problem 3; the shared exclusion class E1 of the §5 hardened theorems); extension to $\mathcal{O}(\delta^2)$ and beyond (Open Problem 1; the natural higher-order extension of Theorem 7.1); Layer 4 axiomatic derivation of Mechanism A from CPP primitive axioms A1–A11 (Open Problem 2; the long-term programme target). None of these is required for the substrate-locality umbrella to hold at sketch-document Layer 3 at the present paper's scope; all three are registered as open problems for the F.1 sub-question's broader closure to higher Layer or extended scope. Two additional Open Problems — Sector-5 schema instantiation (Open Problem 4) and non-vertex-aligned Reading C variants (Open Problem 5) — are research-direction-choosing questions for the broader chirality continuum and Reading C-variant trajectories.

Position in the CPP chirality-from-polytope-geometry long-term programme. The present paper's result is the second sector closure of the broader CPP chirality-from-polytope-geometry programme target (Patch 0528 §14.17; Patch 0539 §15.5). The programme target, in summary, is: *the chirality structure of physical law arises from the discrete geometric primitives of the 600-cell substrate at Reading C, with the substrate primitive $\hat{n}$ as the unifying direction across multiple sectors of physical manifestation.* Capotauro v2.0's spatial-sector closure (manifestations (i)–(iii) at §3) and the present paper's temporal-sector closure (manifestation (iv)) provide two of the anticipated sectoral closures; Open Problem 4 registers the next-sector identification as the natural forward trajectory.

The structural constant $-1/(2\varphi)$ shared between Capotauro v2.0's spatial-sector substrate-locality theorem (§3 of that paper) and the present paper's Theorem 5.3 is not a numerical coincidence; it is the imprint of the same first-shell geometric content (Identity G1 + Identity G2 + the 600-cell host-vertex stabiliser $H_3 = I_h$) operating across both sectors. The chirality continuum's two-sector umbrella structure — spatial + temporal — is anchored at the polytope-geometric level by this shared structural content. The closed-form coefficient $6/\varphi^2$ of Equation (14), derived from $-1/(2\varphi)$ via the icosahedral-sum identity at §7.3, is the temporal-sector specialisation of that shared content.

Forward trajectory. The F.1 sub-question's closure at the present paper's Layer 3 scope opens four trajectories for follow-up work, in approximate order of immediate tractability: (i) G1 publication-grade hardening (Open Problem 3); (ii) substrate-locality umbrella publication-grade hardening (a candidate follow-up Patch beyond the present paper's scope, not registered as a formal Open Problem); (iii) $\mathcal{O}(\delta^2)$ extension (Open Problem 1); (iv) Layer 4 axiomatic derivation of Mechanism A (Open Problem 2). Trajectories (i)–(iii) operate within the present paper's framework qualifiers; trajectory (iv) discharges Mechanism A to the CPP primitive axiom level. Beyond these, the F.2 and F.3 sub-questions of the broader F-series provide additional substrate-direction-primitive sub-questions whose closure is anticipated to follow analogous Layer-3 closure patterns.

The chirality continuum's broader sectoral programme (Open Problem 4 and beyond) and the Reading C-variant programme (Open Problem 5) operate at scope levels above the present paper's framework qualifiers; their closure trajectories are research-direction-choosing rather than derivation, and are registered for future programme-direction Patches in the broader CPP corpus.

The substantive result of the present paper is the temporal-sector substrate-locality theorem; the broader contribution is the demonstration that the 600-cell polytope geometry, at vertex-aligned Reading C with Mechanism A as framework axiom, produces a closed-form physical-content claim at $\mathcal{O}(\delta^1)$ that joins Capotauro v2.0's spatial-sector closure as the second pillar of the chirality continuum's polytope-geometric foundation. The publication-grade rigor at the trio level and the sketch-document Layer 3 closure at the umbrella level together represent the level of substantive contribution targeted by the F.1 trajectory at Patch 0549; the broader programme target (chirality from polytope geometry, with $\hat{n}$ as the unifying direction across multiple sectors) is one sector closer to its long-term completion.

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